Addressing affective dimensions in a community college mathematics preparation course

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ABSTRACT

Attitudes toward mathematics have been shown to be related to mathematics achievement as well as other affective dimensions. An extreme negative attitude toward mathematics encompasses math anxiety characterized by persistent fear, tension, and apprehension that together interfere with performance involving numerical tasks or mathematical problem solving in both ordinary life and academia. This study examined a community college mathematics preparation course that explicitly addressed negative attitudes toward—and reinforced the value of—mathematics to adult students who had voluntarily chosen to commit to this course in order to improve their probability of success in first-year business programs.

Keywords: math anxiety, community college, mathematics preparation course, adult learning, attitude, motivation

Ramirez, Shaw, and Maloney (2018) reported that 25% of university students and up to 80% of community college students reported moderate to high levels of math anxiety. Similarly. Catherine Bruce, Dean of Education at Trent University, found that 80% of adults were math uncomfortable, math averse, or math phobic (C. Bruce, personal communication, April 9, 2009). This problem is so pervasive that some business students choose their college programs intentionally to avoid mathematics, even though business programs such as marketing now focus on quantitative literacy (Bhowmick, Young, Clark, & Bhowmick, 2017). Relatively few studies (e.g., Jamieson, Peters, Greenwood, & Altose, 2016) have examined the relationship between math anxiety and community college students' performance in developmental mathematics programs.

This study addresses and builds upon the research gap identified by Jamieson et al. (2016) and investigates the impact on students of a community college mathematics preparation course offered by the business faculty at a large community college in Ontario, Canada. The course was offered in the summer prior to the students beginning the first year of their business program; enrolment was encouraged but not mandatory. The course was noncredit but included a final exam with a weight of 30% of students' final mark in the course.

Participants in the course were adults aged 20 to 37. All students demonstrated math-phobic or math-uncomfortable attitudes. One of the course's primary goals was to attend to and modify student attitudes while simultaneously addressing mathematics content that was deemed important for success in their first-year business programs.

LITERATURE REVIEW

The following literature review examines current views on math anxiety and attitude towards mathematics.

Attitudes Towards Mathematics

Factors influencing students' attitudes of students toward are complex (Mata, Monteiro, & Peixoto, 2012). In discussing quality teaching of mathematics, Ediger (2012) notes that learners' attitudes: are evaluative and can be presented on a continuum of favourableness; vary in intensity and direction; can be accompanied by or connected with a person's emotions; are relatively durable; are learned and can therefore be taught; and are related to behaviour.

Vandecandelaere, Speybroeck, Vanlaar, Fraine, and Van Damme's (2013) analysis of items in the 2003 Trends in Mathematics and Science Study (TIMSS) identified three dimensions of mathematics attitude: self-confidence, liking mathematics, and usefulness of mathematics. These three dimensions are very similar to a model proposed by Di Martino and Zan (2009) based on qualitative analyses of essay responses of 1,496 Italian students from Grades 2 to 13 on the topic of "Me and Maths." Di Martino and Zan's model also generated three dimensions: perceived competence, emotions towards mathematics, and vision of mathematics.

It is also helpful to examine the origins of students' mathematical attitudes. Hannula (2002) developed a four-phase model of attitude towards mathematics that proposes students evaluate a mathematics task in four sequential phases: (a) an emotional response to the task, typically a quick response based only on emotion in the moment; (b) an associative evaluation, based on similarities to tasks that students have encountered in the past; (c) a competency evaluation, through which students decide whether they feel competent to attempt the task; and (d) evaluation based on students' personal goals, both short term and longer term. Each evaluative phase may return positive or negative results, influencing each student's final decision regarding how to engage with the task.

Lim and Chapman (2013) identify four dimensions of attitude as enjoyment, motivation, self-confidence, and value. The instrument used in this study—Attitudes Towards Mathematics Inventory (ATMI; Tapia & Marsh, 2005)—contains the same four subscales.

A number of correlational studies have identified significant links between attitude and mathematical achievement (e.g., Marchis, 2011; Yaratan & Kasapoglu, 2012). However, such links between attitude and achievement have not been universally supported by research, which in some cases found very weak correlations or no correlations at all (e.g., Di Martino & Zan, 2009; Hannula, 2002). Although Vandecandelaere et al. (2012) state that "attitude towards mathematics is a vital matter in mathematics education" (p. 107), such consensus is all the more surprising considering that the definitions of attitude are very diverse.

Math Anxiety

There are numerous definitions of math anxiety (e.g., Chang & Beilock, 2016; Moran-Soto & Norton, 2018; ; Ramirez, Shaw, et al., 2018; Ramirez, Yang Hooper, Kersting, Ferguson, & Yeager, 2018), though most definitions have common attributes: persistent fear, tension, apprehension, and anxiety that interferes with performance involving numerical tasks or mathematical problem solving in both ordinary life and academia. Such interference may be short term—involving poor mathematics comprehension and low success rates in mathematics courses or situations involving mathematics—or long term, reducing self-confidence, avoidance of mathematics courses, or other disruptive behaviours (Bayazit, 2017). Math anxiety also has been related to reduced mathematics self-efficacy (Bhowmick et al., 2017; Moran-Soto & Norton, 2018); reduced intrinsic motivation with respect to mathematics (Chang & Beilock, 2016); and lack of persistence and learned helplessness with respect to mathematics (Gurefe & Bakalim, 2018), although the direction of causality is disputed.

Math anxiety is identified as a trait-level variable (Ramirez, Shaw, et al., 2018) that is domain-specific (Lauer, Esposito, & Bauer, 2018) and regarded as stable, resilient, and difficult to change (Jamieson et al., 2016). Therefore, strategies to modify math anxiety have been relatively under-researched since it was viewed as being non-malleable. However, research by Jamieson et al.'s (2016) use of the biopsychosocial model of challenge and threat (viewed as a continuum comparing available human resources to situational demands) indicated that math anxiety may be much more malleable than was previously thought, thus offering hope that strategies targeting math anxiety could potentially reduce its impact.

There is substantial research on the direct negative impact of math anxiety on mathematics performance (Chang & Beilock, 2016; Lauer et al., 2018; Ramirez, Yang Hooper, et al., 2018; Soni & Kumari, 2017) as well as indirect negative effects whereby math anxiety negatively impacts mathematics self-efficacy, resulting in lower achievement and potentially learned helplessness in mathematics (Gurefe & Bakalim, 2018). In addition, Ramirez, Shaw, et al. (2018) found an indirect effect in which teachers' level of personal math anxiety negatively influenced the performance of their students and also resulted in students' increased levels of math anxiety. There is debate as to the direction of the causality with math anxiety and academic performance, and Chang and Bielock (2016) found evidence of a reciprocal relationship, in which high math anxiety induced low academic performance, which in turn resulted in increased levels of math anxiety. In a synthesis of over 800 meta analyses of studies on academic performance involving approximately two million students, Hattie (2009) found that math anxiety had a negative effect size of 0.34 with respect to achievement; this means that a student at the 50th percentile under the influence of math anxiety would perform at the 38th percentile.

Substantial research has attempted to identify the factors underlying math anxiety. In their literature review, Garcia-Santillan, Rojas-Kramer, Moreno-Garcia, and Ramos-Hernandez (2017) discussed models with two to six factors. Typical factors included mathematics test anxiety, numerical task anxiety, problem-solving anxiety, and mathematics learning anxiety. Other models included more general traits, such as personal effectiveness, assertiveness, worry, and positive or negative affect towards mathematics. Wang, Shakeshaft, Schofield, and Malanchini (2018) identified two principal factors—learning math anxiety and mathematics exam anxiety—and also linked these two traits to overall mathematics motivation. Wang et al.'s research is discussed in more detail in the Theoretical Framework section below.

The mechanism whereby math anxiety affects performance falls under two categories, which Ramirez, Shaw, et al. (2018) identify as the *disruption account* and the *reduced competence account*. The disruption account is based on cognitive load theory (Sweller, 2011), which posits that math anxiety occupies a portion of working memory, thereby reducing the amount of working memory available for cognition and impairing efficient cognitive processes. The reduced competence account states that math anxiety is the outcome of low math ability, resulting in poor mathematical performance, which in turn fosters math anxiety. Ramirez, Shaw, et al. state that while the disruption account is the generally accepted theory, there is also evidence to support the reduced competence account.

THEORETICAL FRAMEWORK

Two theoretical frameworks informed this study. First, since all the students in this course were at least 20 years old, and some were in their 30s, adult learning models and andragogy were used to frame and evaluate the study. The andragogical principles first formulated by Knowles identify six assumptions about adult learning (Merriam & Bierema, 2014): adults are self-directed; accumulated experience provides a rich resource for learning; adults' readiness to learn is closely related to their social role; there is an immediacy to apply knowledge, thus adults are more problem-centred; adults have high intrinsic motivation and are less driven by extrinsic rewards; and there is a need to know the reason for learning. Based on these concepts, Wlodkowski identified five "pillars" of instruction for adults: expertise, empathy, enthusiasm, clarity, and cultural responsiveness (Wlodkowski & Ginsberg, 2017). From these, Wlodkowski developed a Motivational Framework for Culturally Responsive Teaching, which has four dimensions:

- Establishing inclusion, which includes mutual respect for teacher and learners;
- Developing attitude, fostering a positive disposition toward learning through personal relevance and volition;
- Enhancing meaning through engaging and challenging learning activities that include learners' values and perspectives;
- Engendering competence, reinforcing learners' effectiveness and understanding of something they value.

Wlodkowski detailed 60 strategies that can be employed to support the framework. These strategies were summarized and parsed by Merriam and Bierema (2014) into Wlodkowski's four categories: inclusion, attitude, meaning, and competence. The course outlined in this study was evaluated using Wlodkowski's framework and a comparison against the 60 strategies.

The second theoretical framework attends to student attitudes. Wang et al. (2018) developed a factorial design assessing three attitude dimensions of students: math motivation (MM), learning math anxiety (LMA), and examination math anxiety (EMA). Each dimension is assigned a valence: high, medium, or low. Thus, each student or class can be identified by a vector triple such as (H, L, H) across the three dimensions. Recognizing that both LMA and EMA are negative indicators, the ideal triple would be (H, L, L).

When employing this system to classes, Wang et al. (2018) were able to summarize valences assigned to factors affecting math motivation (self-perceived ability, interest, and importance) as well as factors related to math anxiety (anxiety about math tests, performing math in social situations, performing numerical operations, and learning math concepts). In addition, they demonstrated significant differences in class achievement in mathematics related to the class vector.

This attitude framework was used to characterize each student in the class under study. For example, almost all students were initially characterized as being high in motivation as well as medium to high in exam anxiety. Math anxiety with respect to learning mathematical concepts (LMA) was mixed, and frequently was inversely dependent on the age of the student; that is, the older students were generally less anxious about learning mathematics compared to the younger students who were more recently in formal education. This was a somewhat surprising observation.

METHODOLOGY

This qualitative study used content analysis, which Krippendorff (2013) describes as "a research technique for making replicable and valid inferences from texts (or other meaningful matter) to the contexts of their use" (p. 24). Krippendorff offers a conceptual framework for content analysis that consists of:

- A body of text, the data that a content analyst has available to begin an analytical effort;
- A research question that the analyst seeks to answer by examining the body of text;
- A context of the analyst's choice within which to make sense of the body of text;
- An analytical construct that operationalizes what the analyst knows about the context of the body of text;
- Inferences that are intended to answer the research question, which constitute the basic accomplishment of the content analysis;
- Validating evidence, which is the ultimate justification of the content analysis. (p. 35)

For this study, artifacts examined included the community college course description; course of study; business content; required mathematics content; student comments; observations including verbal and nonverbal responses; student evaluations including quizzes and final examinations; and student responses on the ATMI (Tapia & Marsh, 2005). These multiple sources of data allowed for triangulation of data, providing support for conclusions.

RESEARCH QUESTIONS

This study sought to answer the following research questions through content analysis:

1. How can a mathematics course be structured at the community college level to support success for math-phobic or math-uncomfortable students?
2. What are appropriate measures of success for such a course?

PARTICIPANTS

Participants were 13 adult-aged students (min=20, max=37, M=25.6, SD=7.78) who voluntarily chose to attend a 7-week (42 hour) course to support their mathematics knowledge and skills in anticipation of their first-year business program, which required them to take two mathematics courses including calculus. Although the sample population originally included 14 students, one woman dropped out of the course after the first class. There were nine women and four men: most had been out of school for extended periods, although three students were relatively recent high school graduates; five had received some level of education outside Canada; one student had completed 2 years of a university arts program; two students had not completed high school. Almost all students were married or had dependants, and all had full-time or part-time employment. All students exhibited math-uncomfortable or math-anxious behaviours and expressed low levels of prior knowledge in mathematics. For example, in the first class the students were asked to complete, in groups, a placemat activity in response to the prompt “Tell me everything you know about fractions.” From the entire class, there were two responses: “you can write a fraction as a decimal” and “you need a common denominator.” All had made the commitment to a 2- or 3-year community college business program to enhance their employment status or build an alternative career.

COURSE DESCRIPTION

Because all students were admitted to the business program, all mathematics content was situated within business contexts. No mathematical theory was included and there was an emphasis on the utility of mathematics for the students’ planned programs. Every mathematical topic was introduced through a business problem that could be modeled using mathematics, such as that shown in Figure 1:

> Davina’s company launches an advertising campaign in a town of 30,000 people. The number of people (y) who have heard of the product x days after the campaign begins is given by
>
> $$y = \frac{30000}{1 + e^{-.2(x-15)}}$$
>
> – How many people have heard of the product 1 week after the campaign began?
> – How many people have heard of the product 1 month after the campaign began?
> – What is the maximum number of people who could hear about Davina’s product?
> – Should you consider investing in Davina’s company based on this information?

Figure 1. Sample problem.

The course incorporated technologies (calculators and computer technology) and emphasized a graphical approach. Mathematical models were never presented without a foundation first being developed using graphing technology. Company data for these models were first provided by the instructor, and later provided by students who researched companies of interest to them outside of class time. The practicality of mathematical models was routinely considered and discussed in class and in student groups. Students were encouraged to envision themselves in the role of a company executive. To assist with this, the names of students in the class were always used in the problem or mini-case statements as well as on all questions on the final exam. Since this course was to prepare students for their first-year calculus course, the emphasis was on functions and their derivatives. No plane geometry or trigonometry was included. (Details of the course content are given in Appendix A.) Student groups were used extensively in class, as well as cooperative learning structures such as jigsaw, anticipation guides, concept attainment, think-pair-share, placemat, and dotmocracy (a group method of evaluating alternatives).

Assessment and Evaluation

Formal assessments consisted of six take-home quizzes and a final exam. Technology was permitted for all assessments. The quizzes had group discussion sessions prior to their being submitted to the instructor. The final exam (Appendix B) was done individually, allowed for student choice, and was open-ended in time. Quizzes and the final exam included comics from local newspapers (redacted in this paper due to copyright restrictions). There were also out-of-class assignments, which were formative assessments and not included in the students' grades, and mini case studies that typically were started in class and completed outside of class if necessary. Asynchronous computer discussion chat rooms was available, and students were encouraged to collaborate on these assignments and mini cases either in person or via computer.

How Student Affect Was Addressed

Kunter, Frenzel, Nagy, Baumert, and Pekrun (2011) found that teacher enthusiasm is a key condition for effective instruction and student motivation. Particularly in mathematics, teacher enthusiasm both for mathematics and for teaching were found to be positively correlated with student perceptions of the teacher's support for self-monitoring, providing cognitive challenge, and providing social supports for students (Kunter et al., 2008). More generally, Frenzel, Becker-Kurz, Pekrun, Goetz, and Ludtke's (2018) study of emotional transmission between teachers and students found that teacher displays of emotion and enjoyment, both verbal and nonverbal, positively impacted students' enjoyment and engagement in class, which in turn increased teachers' enjoyment. In this course, the instructor served as a role model who fostered a positive attitude towards the mathematics; engendered respectful interactions with the students and among the students; used think-aloud strategies to model problem-solving techniques; and recognized and celebrated mistakes as learning opportunities. Instructional strategies reinforced the collaborative nature of the course through the use of cooperative learning, group problem solving, and student group problem posing, sometimes using resources that had been brought to class by students.

Patrick, Turner, Meyer, and Midgley (2003) found negative correlations between teacher supportive behaviours and student mathematics avoidance strategies, particularly in the first few days of a course. Students whose teachers espoused an authoritarian classroom atmosphere were more likely to engage in math avoidance. This behaviour is supported by attitude research by Di Martino and Zan (2010), who found that attitudes toward mathematics for most students was negatively influenced by a significant event in their mathematics learning, which they describe as a *disruption event*, almost always related to teacher behaviour. Thus, the instructor in this course was careful to initiate the course as a nonthreatening, collegial experience.

Initially, all students were very reserved, volunteered very few answers, and did not initiate questions or raise issues. Since this was anticipated, the instructor took steps to foster a collaborative and risk-free environment. Student groups were used extensively, and all problems and mini cases featured students' names in the role of company executive or analyst. Leading with a business problem fostered student buy-in and engagement as there was immediate recognition of the value of learning the mathematics (see, e.g., Irvine, 2015). Whole-class discussions rarely requested a response form an individual student, but rather a response from a pair or group, thus creating a less threatening class atmosphere. Students also were encouraged to think of situations from their own experience where the mathematics being learned could have been helpful.

Another instructional strategy was to encourage student problem posing, based on a business article from the newspaper or internet. In this way, students began to see themselves in the role of business consultant rather than student. Newspaper comics were frequently included in student handouts and on lecture slides, as well as on quizzes and the final examination, thus humanizing the mathematics content and reducing stress. As students began to relax more and feel less threatened in class, noticeable changes in behaviour were observed. Students volunteered more responses in class, asked questions for clarification, and raised issues from business that sought mathematical solutions.

A collegial atmosphere was fostered through positive social interactions. Students frequently took their coffee breaks together or in small groups. Because the course was offered in the early evening, a dinner break was scheduled. With student agreement, pairs of students assumed responsibility for providing dinner for the class on a rotation basis. Some students brought home-made food (such as samosas) while others brought store-bought food such as pizzas. The instructor provided dinner approximately once every five classes. Dinners were consumed in the classroom and initially featured social conversations; however, as the course progressed, dinner conversations often continued discussion of the mathematics currently being learned in the class.

Another area in which student affect was explicitly addressed was assessment and evaluation. Prior to take-home quizzes being submitted, there were group discussion sessions of the quiz questions. The availability of discussion relieved much of the tension around assessments and fostered a collegial atmosphere in the class. Prior to the final exam, the instructor provided a topic summary sheet with the categories: "I'm good with this topic"; "I'm OK with this topic"; and "I need to work on this topic" (Appendix A). This gave students the opportunity to critically assess their own knowledge. The topic summary intentionally led with business concepts followed by mathematics content, reinforcing that the course focused on the utility of mathematics in a business context. The format of the final exam, with open-ended time, question choice, use of student names, and judicious use of comics, also served to relieve stress around the evaluation.

ANALYSIS

Two dimensions were evaluated for success: the course itself, and the students. In turn, the course had two major foci: addressing and modifying student attitudes towards mathematics, and emphasizing the value of mathematics for these students' chosen career paths. The

Wlodkowski Motivational Framework for Culturally Responsive Teaching identifies 60 strategies that support the framework's implementation. To evaluate this course, a comparison of the strategies employed in the course showed that 82% of Wlodkowski's strategies were used. Parsing the strategies using the Merriam and Bierema summary found that students in this study used 100% of the inclusion strategies, 70% of the attitude strategies, 85% of the meaning strategies, and 77% of the competence strategies. Three attitude strategies that were not used were learning contracts (due to the compressed time frame for this course); K-W-L (not used in this form but similar strategies were employed); and multiple intelligences.

To evaluate the students' dispositions towards mathematics, the Wang et al. (2018) affective vector framework was used to assign each student a vector triple: Math Motivation (MM), Learning Math Anxiety (LMA), and Exam Math Anxiety (EMA). Assessing student motivation was confounded by two factors: student math motivation, and motivation to take this voluntary course, which presumably represented motivation to succeed in their chosen future business program. Student math motivation was assessed using the ATMI (Tapia & Marsh, 2005), a 40-item questionnaire using a 5-point Likert scale consisting of Strongly Disagree, Disagree, Neutral, Agree, Strongly Agree. It has a high Cronbach's alpha of 0.97 (Asante, 2012). Average scores from the ATMI were quite low (M=2.5, SD=0.282) indicating that most students held negative attitudes towards mathematics. Only three students' average scores exceeded 3.0 (neutral attitude). These students' math motivation scores were therefore coded Medium, while all other students were coded Low.

All students verbally expressed trepidation at facing the final examination. It was difficult to ascertain what portion of this anxiety was due to mathematics (EMA) and what portion reflected general exam anxiety. Several students expressed very negative feelings about examinations based on their recollections of facing exams in high school. Probing by the instructor found that the most negative memories were about math exams. Consequently, all 13 students were coded High for EMA.

LMA was imputed through student behaviours during the 14 classes of this course. While initially all students were quite reserved and infrequently volunteered answers or comments, there was a significant positive change in behaviours as the course progressed. The instructor observed verbal and nonverbal cues indicating increased student engagement and a more positive view of the mathematics content. Increasingly, students not only volunteered responses and opinions in class but also began to bring in relevant materials from work done outside of class. For example, data from companies in which the students had interest was brought into class and used in mathematical modeling; newspaper comics that related to mathematics or business were brought in and sometimes formed the basis for student group problem posing; and issues related to business news items were raised and discussed in a whole-class format, with a view to identifying how mathematics that had been learned in the course could inform business decisions. Thus, while initially all 13 students would have been coded High for LMA, by the end of the course, most would have been rated Medium. Two students were more reluctant to engage in class; this may have been due to continued high LMA, or may have been related to the students' personalities as more introverted.

In summary, the students' affective vector triples (MM, LMA, EMA) were: three students as (M, M, H); two students as (L, H, H); and eight students as (L, M, H). After the final examination was graded, marks were compared to the student affective vectors. All students passed both the final exam and the course. The two (M, M, H) students received the highest marks on the exam and in the course overall; the two (L, H, H) students received the lowest (but still passing) grades on the final exam and the lowest overall grades; and the eight (L, M, H) students' grades were approximately normally distributed between the two extremes, but with a strong negative skewness. While 13 scores are insufficient to draw any statistical conclusions, it does appear that the design and delivery of this course had positive impacts on both student attitudes and student learning.

Content analysis relied heavily on two sources: for student achievement, student submissions (quizzes, assignments, final exam); and for changes in student affect, instructor observations. Instructor observations were particularly useful for assessing student affective changes, since frequently these changes were not verbalized. As mentioned, there was a marked increase in student participation as the course progressed, including responding to instructor questions, seeking clarification, and raising issues. In addition, there was a noticeable change in nonverbal cues. Benzer (2012) notes that during verbal communication, a significant portion of the message is conveyed using nonverbal cues: the content of the speech carries just 7% of the message; tone and quality 38%; and body language 55%. Even without speech, nonverbal behaviours convey affective information: eye contact, facial expressions, gesture, positioning of arms and legs, and posture and ways of sitting all provide information. As Benzer points out, "Verbal messages convey our thoughts whereas nonverbal messages reflect more realistically the inner world of thoughts and feelings" (p. 468). He claims that the use of body language reflects both the students' interest in the material being studied and also their imagination and feelings. In this course, there were noticeable changes in nonverbal behaviours as the course progressed. Initially the majority of students sat with very closed posture; eye contact was limited and often evasive; and facial expressions were blank or uninterested. Later in the course, the opposite reactions were observed; students were much more animated and engaged; open postures were the norm; eye contact and positive facial expressions predominated; and verbal communications conveyed a positive and engaged tone.

Castelli, Carraro, Pavan, Murelli, and Carraro (2012) found that verbal and nonverbal behaviours impact group dynamics. Therefore, when several members of a group or class demonstrate positive affect towards a subject, this positively influenced the other members of the group. This was observed by the instructor as somewhat of a snowball effect in the course; when some students showed a more relaxed and engaged demeanour, others in the class began to imitate those behaviours as well.

DISCUSSION AND IMPLICATIONS

Wlodkowski posits that motivation and learning are inseparable tenets of adult learning, as are learning and culture; motivation is a value-based concept, and intrinsic motivation is key to learning (Wlodkowski & Ginsberg, 2017). These tenets are reflected in his Motivational Framework for Culturally Responsive Teaching, which provides a template for developing mathematical courses for adult learners that focus both on student attitudes and mathematical content. The course described in the current study is a very good fit for Wlodkowski's framework and is especially important since it addresses a major obstacle to adult learning, namely math anxiety. Therefore, the current course serves as a template for the development of similar courses beyond the preparatory level. Paying attention to student affective dimensions is a major consideration in designing mathematics courses, particularly since student affect defines the "important cornerstones of their view of mathematics" (Hannula, 2015, p.269) and the view of mathematics has a significant impact on student cognition and learning (Clarke, 2015).

Schoenfeld (2015) argues convincingly that "people's beliefs/affect/values/preferences/habits of mind shape their in-the-moment decision making" (p.395) and therefore influence both learning and teaching. Thus, when designing a mathematics course, whether a preparation course or a college or university level course, student affect must not only be taken into account but also must be explicitly addressed in course design.

The second research question regarding appropriate measures of success is more problematic. While a course can be evaluated using Wlodkowski's framework, it is also important to have success criteria for students. In the current study, Wang et al. (2018) provided a framework specific to students with mathematics anxiety. However, it is also necessary to include in any evaluation of student success measures of content achievement as well as measures of sustainability. While short-term measures of achievement reflect that these students all achieved a passing grade in this course, a better measure of success would be whether this translated into success in their first-year business mathematics courses and their ultimate success in their chosen business program. Due to confidentiality considerations, it was not possible to track these students' progress through first year and beyond. Future studies of similar courses that address student affective dimensions should build into their evaluation structures measures that reflect these longer-term goals.

An additional success criterion should reflect positive modifications to student beliefs or attitudes towards mathematics, although Schoenfeld (2015) states that these constructs are very resilient and true change may take months or years. Such far-ranging outcomes are likely impossible to track. However, positive changes in these constructs still represents a goal of education and a measure of the successfulness of any mathematics course.

LIMITATIONS

This was primarily a qualitative study, with a small number of students (13) and a restricted time frame (7 weeks). Thus, no generalization of results is feasible. However, it seems clear that for this specific group of students, positive results were obtained both in mathematical content acquisition and in attitude changes. It could be conjectured that the changes in attitude, if maintained, will have significantly greater impact on these students' careers. Additional research studies of similar courses would need to be conducted to support claims of replicability of outcomes.

REFERENCES

Asante, K. O. (2012). Secondary students' attitudes towards mathematics. *IFE PsychologIA*, *20*(1), 121-133.

Bayazit, Y. (2017). Anxiety of students preparing for the university entrance exam towards math lessons. *Pedagogy*, *89*(5), 650-664.

Benzer, A. (2012). Teachers' opinions about the use of body language. *Education*, *132*(3), 467-473.

Bhowmick, S., Young, J., Clark, P., & Bhowmick, N. (2017). Marketing students' mathematics performance: The mediating role of math anxiety on math self-concept and math self-efficacy. *Journal of Higher Education Theory and Practice*, *17*(9), 104-117.

Castelli, L., Carraro, L., Pavan, G., Murelli, E., & Carraro, A. (2012). The power of the unsaid: The influence of nonverbal cues on implicit attitudes. *Journal of Applied Psychology*, *42*(6), 1376-1393. doi:10.1111/j.1559-1816.2012.00903.x

Chang, H., & Beilock, S. (2016). The math anxiety-math performance link and its relation to individual and environmental factors: A review of current behavioral and psychophysiological research. *Current Opinions in Behavioral Sciences*, *10*, 33-38. doi:10.1016/j.cobeha.2016.04.011

Clarke, D. (2015). Reaction to section 1: Faith, hope and Charity: Theoretical lenses on affect. In B. Pepin & B. Roesken-Winter (Eds) *From beliefs to dynamic affect systems in mathematics education* (pp. 119-134). Switzerland: Springer International.

Di Martino, P., & Zan, R. (2010). "Me and maths": Towards a definition of attitude grounded in students' narratives. *Journal of Mathematics Teacher Education*, *13*(1), 27-48. doi:10.1007/s10857-009-9134-z

Ediger, M. (2012). Quality teaching in mathematics. *Education*, *133*(2), 235-238.

Frenzel. A., Becker-Kurz, B., Pekrun, R., Goetz, T., & Ludtke, O. (2018). Emotion transmission in the classroom revisited: A reciprocal effects model of teacher and student enjoyment. *Journal of Educational Psychology*, *110*(5), 628-639. doi:10.1037/edu0000228

Garcia-Santillan, A., Rojas-Kramer, C., Moreno-Garcia, E., & Ramos-Hernandez, J. (2017). Mathematics test, numerical task and mathematics course as determinants of anxiety toward math on college students. *European Journal of Contemporary Education*, *6*(2), 240-253. doi:10.13187/ejced.2017.2.240

Gurefe, N., & Bakalim, O. (2018). Mathematics anxiety, perceived mathematics self-efficacy and learned helplessness in mathematics in faculty of education students. *International Online Journal of Educational Sciences*, *10*(3), 147-161. doi:10.15345/iojes.2018.03.010

Hannula, M. (2002). Attitude towards mathematics: Emotions, expectations and values. *Educational Studies in Mathematics*, *49*, 25-46. doi:10.1023/A:1016048823497

Hannula, M. . (2015). Reaction to section 2: The relevance of affective systems and social factors: A commentary. In B. Pepin & B. Roesken-Winter (Eds) *From beliefs to dynamic affect systems in mathematics education* (pp. 269-280). Switzerland: Springer International.

Hattie, J. (2009). *Visible learning: A synthesis of over 800 meta-analyses relating to achievement.* London, UK: Routledge.

Irvine, J. (2015). Problem solving as motivation in mathematics: Just in time teaching. *Journal of Mathematical Sciences*, *2*, 106-117.

Jamieson, J., Peters, B., Greenwood, E., & Altose, A. (2016). Reappraising stress arousal improves performance and reduces evaluation anxiety in classroom exam situations. *Social Psychological and Personality Science*, *1*(9). doi:10.1177/19485506/6644656

Krippendorff, K. (2013). *Content analysis: An introduction to its methodology* (3rd ed.). Thousand Oaks, CA: Sage.

Kunter, M., Frenzel, A., Nagy, G., Baumert, J., & Pekrun, R. (2011). Teacher enthusiasm: Dimensionality and context specificity. *Contemporary Educational Psychology*, *36*(4), 289-301. doi:10.1016/j.cedpsych.2011.07.001

Kunter, M., Tsai, Y., Klusmann, U., Brunner, M., Krauss, S., & Baumert, J. (2008). Students' and mathematics teachers' perceptions of teacher enthusiasm and instruction. *Learning and Instruction*, *18*(5), 468-482. doi:10.1016/j.learninstruc.2008.06.008

Lauer, J., Esposito, A., & Bauer, P. (2018). Domain-specific anxiety relates to children's math and spatial performance. *Developmental Psychology*, *54*(11), 2126-2138. doi:10.1037/dev0000605

Lim, S. Y., & Chapman, E. (2013). Development of a short form of the attitudes toward mathematics inventory. *Educational Studies in Mathematics*, *82*(1), 145-164. doi:10.1007/s10649-012-9414-x

Marchis, J. (2011). Factors that influence secondary school students' attitude to mathematics. *Procedia – Social and Behavioral Sciences*, *29*, 786-793. doi:10.1016/j.sbspro.2011.11.306

Mata, M., Monteiro, V., & Peixoto, F. (2012). Attitudes towards mathematics: Effects of individual, motivational, and social support factors. *Child Development Research*, *2012*, 1-10. doi:10.1155/2012/876028

Merriam, S., & Bierema, L. (2014). *Adult learning: Linking theory and practice*. San Francisco, CA: Jossey-Bass.

Moran-Soto, G., & Norton, P. (2018). Effects of an intensive remedial math course on engineering students' math anxiety and math self-efficacy. In T. Hodges, G. Roy, & A. Timinski (Eds), *Proceedings of the 40th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 974-977). Greenville, SC: University of South Carolina & Clemson University.

Patrick, H., Turner, J., Meyer, D., & Midgley, C. (2003). How teachers establish psychological environments during the first days of school: Associations with avoidance in mathematics. *Teachers College Record*, *105*(8), 1521-1558. doi:10.1111/1467-9620.00299

Ramirez, G., Shaw, S., & Maloney, E. (2018). Math anxiety: Past research, promising interventions, and a new interpretation framework. *Educational Psychologist*, *53*(3), 145-164. doi.10.1080/00461520.2018.1447384

Ramirez, G., Yang Hooper, S., Kersting, N., Ferguson, R., & Yeager, D. (2018). Teacher math anxiety relates to adolescent students' math achievement. *AERA Open*, *4*(1), 1-13. doi:10.1177/2332858418756052

Schoenfeld, A. (2015). What counts, when? Reflections on beliefs, affect, attitude, orientations, habits of mind, grain size, time scale, context, theory, and method. In B. Pepin & B. Roesken-Winter (Eds) *From beliefs to dynamic affect systems in mathematics education* (pp. 395-403.). Switzerland: Springer International.

Soni, A., & Kumari, S. (2017). The role of parental math anxiety and math attitude in their children's math achievement. *International Journal of Science and Mathematics Education, 15*, 331-347. doi:10.1007/s10763-015-9687-5

Sweller, J. (2011). Cognitive load theory. In J. P. Mestre & B. H. Ross (Eds.), *The psychology of learning and motivation (Vol. 55): Cognition in education* (pp. 37-76). San Diego, CA: Elsevier. doi:10.1016/B978-0-12-387691-1.00002-8

Tapia, M., & Marsh, G. E. (2005). Attitudes toward mathematics inventory redux. *Academic Exchange Quarterly*, *9*(3), 272-275.

Vandecandelaere, M., Speybroeck, S., Vanlaar, G., Fraine, B., & Van Damme, J. (2012). Learning environment and students' mathematics attitude. *Studies in Educational Evaluation*, *38*(3-4), 107-120. doi:10.1016/j.stueduc.2012.09.001

Wang, Z., Shakeshaft, N., Schofield, K., & Melanchini, M. (2018). Anxiety is not enough to drive me away: A latent profile analysis on math anxiety and math motivation. *PLOS One*, *13*(2). doi:10.1371/journal.pone.0192072

Wlodkowski, R., & Ginsberg, M. (2017). *Enhancing adult motivation to learn*. San Francisco, CA: Jossey-Bass.

Yaratan, H., & Kaspoglu, L. (2012). Eighth grade students' attitude, anxiety, and achievement pertaining to mathematics lessons. *Procedia – Social and Behavioral Sciences*, *46*, 162-171. doi:10.1016/j.sbspro.2012.05.087

APPENDIX A: STUDENT-FRIENDLY COURSE OF STUDY

Ready for The End Checkup

Topic	Concepts	I'm good with this topic	I'm OK with this topic	I need to work on this topic
Financial math	Percent change			
	Ratio analysis (horizontal/vertical)			
	Simple interest			
	Compound interest--final amount --present value --find rate --find time (e.g. doubling time) Annuities--future value of an annuity due --present value of an ordinary Annuity --finding a loan payment --finding a mortgage payment Finding the future value of a series of unequal payments or with different interest rates			
Business concepts	Revenue			
	Cost (fixed/variable)			
	Profit			
	Breakeven point			
	Marginal cost			
	Marginal revenue			
	Marginal profit			
	Supply and demand			
	Elasticity of demand			
	Elasticity of cost			
	Elasticity of supply			
	Equilibrium point			
	Inelastic demand			
	Elastic demand			
	Production intervals to make a profit			
	Product pricing (using a counter)			
Algebra	Power laws			
	Negative and zero exponents			
	Fractional exponents			
	Factoring a quadratic equation			
	Quadratic formula			
	Factor theorem			
	Long division with algebra			
	Logarithms --what are they --conversions between log notation and			

	exponential notation --log laws --solving exponential equations			
Linear relations	$y = mx + b$			
	Slope			
	y-intercept			
	x-intercept			
	Finding the equation of a line			
	Parallel lines			
	Perpendicular lines			
	Equations of horizontal lines			
	Equations of vertical lines			
	Graphing a line using intercepts			
Functions	Function notation $f(x)$			
	Vertical line test			
	Domain			
	Range			
	Magnificent 7 functions			
	Average value of a function			
Quadratic functions	$y = ax^2 + bx + c$			
	Graph (parabola)			
	Vertex			
	Axis of symmetry			
	Direction of opening			
	Maximum or minimum			
	y-intercept			
	Zeros {(x-intercepts)			
	Domain			
	Range			
Polynomial functions	e.g. $y = ax^3 + bx^2 + cx + d$			
	y-intercept			
	Zeros			
	General shape (N or W)			
	Domain			
	Range			
Rational Functions	$y = \frac{k}{x} \; or \; y = \frac{k}{x-a}$			
	Intercepts			
	Domain			
	Range			
	Asymptotes			
Exponential functions	$y = a^x$			
	Domain			
	Range			

	Intercepts			
	Asymptotes			
	$y = e^x$			
	Domain			
	Range			
	Intercepts			
	Asymptotes			
Logistic function	$y = \frac{L}{1 + ae^{-kx}}$			
	Limit			
	Domain			
	Range			
	Intercepts			
	Asymptotes			
Logarithmic function	$y = \ln(x)$			
	Domain			
	Range			
	Intercepts			
	Asymptotes			
Calculus	Slope of tangent			
	Equation of tangent			
	Finding max/min using derivatives (slopes)			
	Average rate of change			
	Instantaneous rate of change (marginal)			
	Power rule $y' = nx^{n-1}$			
	Product rule $y' = f'g + g'f$			
	Quotient rule $y' = \frac{f'g - g'f}{g^2}$			
	Chain rule $y' = \frac{df}{dg} \times \frac{dg}{dx}$			
	Derivative of $y = e^x$ $y' = e^x$			
	Derivative of a logistic function $y = \frac{L}{1 + ae^{-kx}}$			
	Derivative of $y = \ln(x)$ $y' = \frac{1}{x}$			
	Derivative of $y = a^x$ $y' = a^x(\ln a)$			
	Second derivative			
	Local and absolute extrema			
	Finding maxima and minima using the first			

	derivative			
	Classifying maxima and minima using the second derivative test			
	Finding points of inflection			
Confidence in the concepts from this course	How confident are you that you are OK with the concepts from this course			
	How confident are you that you are OK with the skills and procedures from this course			
	How confident are you that you will be OK with the math course in September			

APPENDIX B: FINAL EXAM

Give complete solutions to ANY 10 of the following questions. Be sure to clearly indicate the question number for each question you do. If you do more than 10 questions, the first 10 solutions will be marked, unless you indicate otherwise. Enjoy yourself!

1. Davina just bought a new motorcycle for $10,000. The value of the motorcycle depreciates over time according to the model $V(t) = 10000e^{-\frac{t}{4}}$ where t is the time in years.
   a) At what rate is the value of the motorcycle depreciating the instant Davina drives it off the dealer's lot?
   (b) Davina decides that she will stop insurance coverage for collision once the motorcycle has depreciated to ¼ of its initial value. When will she drop the insurance coverage?
   (c) At what rate is the motorcycle depreciating when Davina drops the insurance coverage?
2. As manager of an 80-unit apartment complex, Stefan is trying to decide what to charge his tenants for rent. He knows from past experience that at a monthly rent of $1000, all the units will be full. However, one additional unit will remain vacant for every $50 increase in rent. What rent should Stefan charge to maximize revenue from the apartment complex? What is that maximum revenue?
3. As company accountant, Youseff determined that the cost to produce x thousand items can be modeled by $C(x) = 1000 + x^2 + \frac{16000}{x}$
   (a) How many items should the company produce to minimize costs?
   (b) What is the marginal cost of producing the 101st item?
   (c) How fast is the marginal cost changing at that point?
   (d) How can your answer to (c) be used to make a business decision?
4. Geraldine plans to invest $4000, part in a "safe" investment earning 2% and the rest in a "risky" investment earning 10%. She wants her investment to earn at least $280 interest in the first year.
   (a) How much should Geraldine invest at each rate?
   (b) If her investments earn interest compounded annually, what will her total investment be worth 5 years from now?
5. Marec's internet company sales this year are $50000. Five years from now, sales are forecast to be $180,000.
   (a) Find an exponential model for Marec 's sales in the form $S(t) = S_0e^{kt}$
   (b) Use your model to forecast sales for Marec 's company 10 years from today.
   (c) Is an exponential model realistic for Marec? Explain.
6. Gorodo's business began making a profit the first year it opened. Her profit function (in $thousands) when selling x thousand units is $P(x) = \frac{4x}{3x^2+243}$
   (a) What is Gorodo 's marginal profit when she sells 500 units?
   (b) What level of sales will result in maximum profit?
   (c) What is her maximum profit?
   (d) How fast is Gorodo 's marginal profit changing when she sells 10000 units?
   (e) What does your answer to (d) tell us?

7. The percentage of television viewers who remember what product is regularly advertised on the television show Survivor is $N(x) = 70 - 30e^{-.2x}$ where x is the number of times the viewer has seen the show.
   (a) What percentage of viewers remember the product if they have seen the show 5 times?
   (b) How fast is the percentage of viewers who remember the product changing for viewers who have seen the show 6 times?
   (c) The advertisers wants to know how many times viewers must see the show to result in 50% of the viewers remembering the product. Answer their question for them.
8. Nadine's company produces email advertising flyers and sends them to targeted market segments. She estimates their advertising "reach", i.e. the number of people in the target market segment who will read the email before trashing it, to be $N(x) = \frac{75000}{1+e^{-0.2x}}$ where x is the number of repeats of the same email message.
   (a) What is the number of people in the target market segment?
   (b) How many people will be reached if the email is repeated 8 times?
   (c) What is the marginal increase in targeted people reached after the email is repeated 5 times?
   (d) Do you expect this marginal rate to increase or decrease as the number of email repeats grows very large? Why?
9. Tumic estimates her business's profit function (in $thousands) to be $P(x) = x^3 - 3x^2 - 4x + 12$ where x is thousands of units. Her factory's capacity is 10,000 units.
   (a) What levels of production are breakeven points for Tumic?
   (b) For what intervals of production is Tumic making a profit?
   (c) What level of production gives Tumic her maximum profit?
   (d) What is her maximum profit?
10. Steve's honey company costs are given by $C(x) = -2000x + 18000x^{\frac{2}{3}}$ where x is the number of kg of honey produced, in thousands.
    (a) Find the production level that gives Steve minimum cost.
    (b) What is Steve's minimum cost?
    (c) What is Steve's marginal cost of production at a production level of 3000 kg of honey?
    (d) What level of production will give a marginal cost of $4?
11. Martine has a fast-growing company that sells advertising space on mobile media, like balloons, T-shirts, and portable banners. She estimates her profit function to be $P(x) = x^2e^{0.1x} + 10x^{\frac{3}{4}}$ where x is thousands of units sold
    (a) What is her profit when she sells 10,000 units?
    (b) What is her marginal profit when she sells 10,000 units?
    (c) How fast is her marginal profit growing when she sells 10,000 units?
    (d) Comment on whether this is a realistic profit model for Martine's business.
12. Sala has decided to buy a fish farm. He will raise fish and sell them to supermarkets and restaurants when they are large enough. The length (cm) of a certain species of fish, x months after it hatches, is given by $L(x) = 3x + 12\ln(5x)$
    (a) How long is a fish after 2 months?

(b) How fast is the fish growing after 3 months?
(c) Sala plans to harvest and sell the fish when they reach a length of at least 50 cm. When can Sala harvest the fish?
(d) Suppose Sala decides to wait to harvest his fish until the fish's growth rate is 1 cm/month. Explain why this is a poor decision.

13. Rachel's new fashion company has a profit function (in $ thousands) given by $P(x) = \frac{2000}{x} + \frac{25x^2}{\sqrt{(5x+100)}}$ where x is sales in thousands of units.
(a) What is Rachel's marginal profit when her company sells 10,000 units?
(b) At what rate is Rachel's marginal profit changing when her company sells 10,000 units?
(c) What is Rachel a's average profit when she sells 10,000 units?
(d) Comment on this profit model. Does this profit function predict that Rachel's company will be a success or a failure? Why?