Decision models for the newsvendor problem – learning cases for business analytics

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ABSTRACT

Single-period inventory models with uncertain demand are very well known in the business analytics community. Typically, such models are rule-based functions, or sets of functions, of one decision variable (order quantity) and one random variable (demand). In academics, the models are taught selectively and usually not completely. Students are exposed to applications of selected models and solution aspects usually within the Inventory Control or similar topics. This learning-case oriented paper attempts to provide a fuller coverage of the solution process, starting with mathematical model building, following with calculus-based solution development and ending with an alternative approach through simulation. A special emphasis is dedicated to the learning processes, providing students with a variety of tools selected from Algebra, Calculus, Probability, Statistics and Spreadsheet Technology. The students are expected to benefit from studying this paper to better understand business solving processes. All computational operations are implemented in a spreadsheet program.

Keywords: model, optimization, probability, inventory, simulation, spreadsheet.

PROBLEM STATEMENT

A classical single time-period inventory model is also known as the Newsvendor, News Boy, or Perishable model (Wikipedia- Newsvendor, 2018). Using such a model, a decision analyst attempts to optimize a simple inventory decision for a situation in which a seller acquires a bundle of newspapers at the beginning of a single inventory period to sell the newspapers during this period. Any inventory remaining at the end of the period is sold at a scrap price. Suppose that the seller purchases the newspapers for c = \$0.40 each and sells them for p = \$1.00 each. The surplus newspapers, if any, are sold for scrap at s = \$0.10 each.

Using a profit-oriented approach, the seller's payoff function can be defined as follows:

$$Payoff(Q,D) = \begin{cases} (p-c)Q & if \quad Q \le X \\ pX - cQ + s(Q-X) & otherwise \end{cases} \quad (1)$$

If demand X is at least as high as supply Q, all newspapers will be sold and the payoff will be the difference between revenue $p \cdot Q$ and purchasing cost $c \cdot Q$. On the other hand, if demand X is below supply Q, there will be (Q-X) unsold newspapers. In this case, revenue = $p \cdot X + s \cdot (Q-X)$ and cost = $c \cdot Q$. Formula 1 provides a precise definition for the payoff depending on Q and X, where Q is a decision (controlled) variable and X is a random (uncontrolled) variable. The optimal solution for Q (supply) is a value, Q^*, that maximizes the expected payoff:

$$Q^* : E[Payoff(Q^*, X)] = \underset{Q}{Max}\{E[Payoff(Q,X)]\} \quad (2)$$

The same solution can be developed, using a cost-based function, taking into account two types of cost components: a cost of underestimating demand and a cost of overestimating demand:

$$Cost(Q,X) = \begin{cases} c_o(Q-X) & if \quad X \le Q \\ c_u(X-Q) & otherwise \end{cases} \quad (3)$$

Parameters c_u and c_o represent per unit cost of underestimating and overestimating demand, respectively, where $c_u = (p-c)$ and $c_o = (c-s)$. Applying this approach, the optimal solution for Q (supply) is a value, Q^*, that minimizes the expected cost:

$$Q^* : E[Cost(Q^*, X)] = \underset{Q}{Min}\{E[Cost(Q,X)]\} \quad (4)$$

It is interesting to note that parameters c_u and c_o can also be interpreted as a marginal gain and loss, respectively. Using such parameters, the payoff function can also be defined as:

$$Payoff(Q,X) = Min(Q, X)^* c_u - Max(0, Q - X)^* c_o \quad (5)$$

Optimization criteria (2) and (4) can be used only if the probability distribution of demand, $F(x) = P(X \le x)$, is known. Otherwise decision about supply Q can be made only with respect to a strategy adopted by the analyst.

This paper shows six different methods for solving the Newspaper Seller Problem. The first three method are based on the Decision Theory (*MaxiMax*, *MaxiMin*, and *MiniMax Regret*). These methods assume minimal information about demand (X). The analyst is only aware of the range of demand levels. The probability distribution of demand is unknown. The last three methods assume that the probability distribution of demand X is known. These methods represent three different implementations for the cost-based optimization criterion (4). The first one assumes that demand X is a discrete random variable. The second one uses a Calculus derivation and the third one —simulation algorithm in order to find the optimal solution.

LEARNING OBJECTIVES

In academia, learning formal methods for business problem solving is usually part of Management Science or Operations Management, generally known as Quantitative Methods (QM). Currently, many authors refer to these domains as Business Analytics. Others explore QM problems within the so called Data Science field. In any case, the topical structure of these domains is typically method or model oriented. Students learn how to solve problems using Statistical Models, Linear Programming, Game Theory, Inventory Models, Queuing Models, etc. As they move from one topic to another, they are exposed to different problems. This paper uses the same business problem to show different models and solution methods in situations with different amount and type of information available about one of the problem variables. Here, this variable is demand, a random variable (*X*).

As mentioned, six cases for solving the single-period inventory problems with uncertain demand are presented, providing the students with different model building and solution methods. All problem solving procedures, with exception to the Calculus based, utilize spreadsheet data structures and tools.

Upon completing the cases, the student will have better understanding of method selection necessary for solving the same business problem, depending on the level of awareness about the state of the nature. Since the students deal with the same problem, after going through the first case, they already have a good understating of the problem. While dealing with the subsequent cases, they can focus more on the solution methods. This problem-oriented learning process should help the students gain deeper insights into the world of decision problem modeling and optimization techniques.

DECISION MAKING UNDER UNCERTAINTY

The three cases, presented in this section, assume that random demand *X* is uncertain and known to vary between some minimum and maximum value. Making a decision about variable *Q* (newspaper supply) falls into the domain of decision making under uncertainty within the Decision Theory. The decision maker, referred to as *player*, plays against the unpredictable nature—the states of newspaper demand (*X*). The three models are well described theoretically in (Anderson, et al. 1995, p. 110-13).

The future states of nature (levels of demand) are totally unpredictable. It is only know which states may occur. It is not known how likely the states are. Making decision under such conditions is sort of a guessing. However, it is important to be aware about possible options that are available for all the states of the nature. In selecting the *right* decision, one can apply one of the following approaches: *MaxiMax*, *MaxiMin*, or *MiniMax Regret*. All these approaches represent different decision making *attitudes*. (*Optimistic*, *Conservative*, *Neutral*).

CASE 1: OPTIMISTIC APPROACH – *MAXIMAX*

Suppose that there are *m* alternative decisions (q_i, i=1,2,…,*m*) and *n* unpredictable outcomes of the state of nature (x_j, j=1,2,…,*n*). The optimistic strategy suggests selection of that alternative decision (q_k) which yields the highest payoff.

$$q_k : \underset{i}{Max}\left\{\underset{j}{Max}\left\{Payoff\left(q_i, x_j\right)\right\}\right\},\ \ j = 1,2,\cdots,n;\ \ i = 1,2,\cdots,m \tag{6}$$

The *optimistic decision* is determined by first calculating the highest possible payoffs for each alternative decision over the entire domain of the states of the nature. The *MaxiMax* solution is pointed to by the payoff, having the maximum value.

Figure 1 shows a spreadsheet based implementation of this strategy. According to this strategy, the best alternative is $Q = q_7 = 140$ that will yield the payoff of $84. This payoff is guaranteed only for the maximum value of demand, $X = x_7 = 140$. Thus the *MaxiMax* solution is determined as *Payoff* (q_7, x_7) = *Payoff* (140, 140) = $84.

The following names (named ranges), formulas and operations are used to create the spreadsheet (notice that fixed labels and formats are shown in Figure 1):

Names:

```
E3        _c
E4        p
E5        s
C08:I08   x
B09:B15   q
C09:I15   payoff
J09:J15   maxPayoff
J16       maxiMaxPayoff
```

Formulas:

```
C09:I15   {=IF(q<=x,(p-_c)*q,p*x-_c*q+s*(q-x))}                (7)
J09:J09   =MAX(C09:I08)
```

Operations:

1 To enter the payoff formulas, select range `C09:I15`, type the following formula: `=IF(Q<=D,(p-_c)*Q,p*D-_c*Q+s*(Q-D))` and press simultaneously `Shift+Ctrl+Enter`

2 Copy `J09` and paste to `J10:J15`.

Formulas:

```
J16       =MAX(maxPayoff)
J17       =INDEX(q,MATCH(J16,J09:J15,0))
```

Figure 1 shows an Excel implementation of this model, using the above setup. Notice that cell E3 is named as **_c.** It represents the purchasing cost shown in the model as ***c***. Excel has modified this name since **c** is a reserved keyword.

This case is an excellent opportunity for the student to learn and reinforce their spreadsheet skills, regarding development of mathematical modeling. In particular, they learn more about using named cells and ranges in formulas in which full and partial absolute references should be used. The main formula, implementing the payoff function (1), uses the

`IF()` function entered directly into the entire payoff range (`C09:I15`). It is a tricky way to replicate spreadsheet formulas. Some student have hard time understanding and doing it. The instructor should demonstrate this technique several times, especially if the students have never developed such formulas.

Extra attention should also be paid to the structure of the `IF()` function. It may be advantageous to break down this function into two separate formulas:

`=(pp-pc)*vQ` (8)

`=pp*vD-pc*vQ+ps*(vQ-vD)` (9)

The first formula can be entered along and above the diagonal cells in the payoff range (`C10:I16`) and the second one—below the diagonal. Although this approach does not use the `IF()` function, it is much more laborious. In fact, the first formula has to be entered in the upper-left corner of the range (the first diagonal cell) as `=(p-_c)*$B23`. Next, it has to be copied and pasted gradually to all other cells along and above the diagonal cells. Figure 2 shows the spreadsheet implementations of the two formulas.

There is another way to express the *Payoff* formula in Excel. It is based on function (5). Unfortunately, Excel can't properly express the *Min*() and *Max*() functions using the named ranges. The following formula would have to be entered in cell C9 and copy-pasted to the other cells of the payoff range:

`=MIN(C$8,$B9)*(p-_c)-MAX(0,$B9-C$8)*(_c-s)` (10)

This formula is here equivalent to the `IF()` formula (8). It would a good exercise to ask the student to apply this formula and to explain why the two formulas produce the same result?

This optimistic approach (expecting that the nature will act in our favor) provides the payoff of $84 when the order quantity is picked at the highest level, Q = 140. The worst-case scenario that may occur is *Payoff* = $30, when demand ($X$) gets to the lowest level of 80.

This model is available as an Excel workbook, 1-MaxiMax.xlsx, stored in ZIP archive spim2018letkowski.zip (Letkowski, 2018).

CASE 2: CONSERVATIVE APPROACH: *MAXIMIN*

The conservative approach represents a *greedy pessimism*. It first finds the worst payoff for each decision option. Then it picks up that option which has the highest payoff among the worst ones. Thus, this approach employs the *MaxiMin* strategy:

$$q_k : \underset{i}{Max}\left\{\underset{j}{Min}\left\{Payoff\left(q_i, x_j\right)\right\}\right\}, \;\; j=1,2,\cdots,n; \;\; i=1,2,\cdots,m \qquad (11)$$

Figure 3 shows a spreadsheet based implementation of this strategy. As this model is very similar to the previous one, the *Payoff* formulas are the same. The only differences are:

Names:

```
J09:J15  minPayoff
J16      maxiMinPayoff
```

Formulas:

```
J09      =MIN(C10:I10)
J16      =MAX(minPayoff)
J17      =INDEX(q,MATCH(maxiMinPayoff,minPayoff,0))
```

The students can recycle the previous model. Structurally, the models are very similar. The only, significant difference is in the *MinPayoff* (range `J09:J15`) column where previous `Max()` function is replaced by the `Min()` function. As shown above, some of the cells (ranges) are renamed for semantic reason.

According to this strategy, the best alternative is $Q = q_1 = 80$ that yields the payoff of $48, even if the worst state of nature (demand level) happens. So this solution guarantees the payoff to be $48, way below the *MaxiMax* result ($84) but, again, the latter may actually happen. Thus the *MaxiMin* solution is determined as $Payoff\,(q_1, \cdot) = Payoff\,(80, \cdot) = \$48$ (where symbol · means any value selected from feasible options, here from x_1, x_2, … , x_n).

This model is available as an Excel workbook, 2-MaxiMin.xlsx, stored in ZIP archive spim2018letkowski.zip (Letkowski, 2018).

CASE 3: OPPORTUNITY LOSS APPROACH: *MINI-MAX REGRET*

The *MiniMax Regret* strategy (known also as the opportunity loss strategy) is appropriate for those decision makers that avoid extreme strategies. It is a sort of a compromise between the optimistic approach and the conservative approach. It is based on as regret table. For each decision, we find the highest regret. The decision to be selected, q_k, is the one that has the lowest regret value (thus *MiniMax Regret*):

$$q_k : MaxRegret(q_k) = \underset{i}{Min}\left\{\underset{j}{Max}\left\{Max\left\{Payoff\left(q_i, x_j\right)\right\} - Payoff\left(q_i, x_j\right)\right\}\right\},\quad i = 1,2,\cdots,m\,,\ j = 1,2,\cdots,n \qquad (12)$$

To generate a payoff table for this strategy, first regret values must computed. For each supply level q_i and demand level x_j, the difference between the maximum payoff for supply q_i , $Max(Payoff(q_i,x_.)$, a column maximum, and the payoff value for each supply, $Payoff(q_i,x_j)$, is calculated. This value represent a payoff loss of the given decision, q_i, compared to the maximum possible payoff. Indexes i, j represent row and column numbers, respectively. The following names (named ranges), formulas and operations are used to create the spreadsheet:

Names:

```
E3       _c
E4       p
E5       s
C08:I08  x
B09:B15  q
C09:I15  payOff
C16:I16  maxPayoff
C21:I27  regret
J21:J27  maxRegret
J28      minMaxRegret
```

Formulas:

```
C12:I18   {=IF(q<=x,(p-_c)*q,p*x-_c*q+s*(q-x))}
C16:I16   =MAX(C09:C15)
```

Operations:

1 To enter the payoff formulas, select range `C09:I15`, type the following formula:
`=IF(Q<=D,(p-_c)*Q,p*D-_c*Q+s*(Q-D))`
and press simultaneously `Shift+Ctrl+Enter`

2 Copy `C16` and paste it to `D16:I16`.

Formulas:

```
C21:I27 {=maxPayoff-payOff}
```

Operations:

3 To enter the regret formulas, select range `C21:I27`, type the following formula:
`=maxPayoff-payOff`
and press simultaneously `Shift+Ctrl+Enter`

Formulas:

```
J20        =MAX(maxPayoff)
J21        =INDEX(q,MATCH(J18,J10:J16,0))
```

This setup generates a spreadsheet model as shown in Figure 3.

According to this strategy, the best decision alternative is Q = 120 (q_5) that provides the smallest among the highest possible regrets $12.

This model is available as an Excel workbook, 3-MiniMaxRegret.xlsx, stored in ZIP archive spim2018letkowski.zip (Letkowski, 2018).

PROBABILISTIC DECISION MODELS

The remaining models are structured, assuming the probability distribution of demand is known. There cases are presented. The first case (**Case 4**) is dedicated to situations where demand X is a discrete random variable with a known empirical distribution. The following two cases assume that X is a continuous variable also with known (Normal) distribution.

Case 4: A Newsvendor Numeric Model with a Probabilistic, Discrete Demand

The probability distribution represents the richest level of awareness about uncertain events or processes. It allows us to specify a precise optimization criterion based on the concept of the expected value based on criteria (2) and (4). Each decision q_i has its expected (mean) value defined as follows:

$$E[Payoff(q_i, X)] = \sum_{j=1}^{n} Payoff(q_i, x_i)p(x_i),\ i = 1,2,\cdots,m \qquad (13)$$

Probability function, $p(q_i) = P(Q = q_i)$, is defined as an array, $p_1, p_2, \ldots, p_n$ of probabilities assigned to discrete demand levels of demand, $x_1, x_2, \ldots, x_n$. Figure 4 shows this distribution as a histogram. It is based on the following probability distribution of demand X: `{(x,p)}=` `{(70,0.02),(80,0.1),(90,0.22 ),(100,0.32),(110,0.22),(120,0.1),(130, 0.02)}.`

The spreadsheet model for this case can be developed, starting with, for example, the *MaxiMax* model. A handful of name and formula changes will produce a model as shown in Figure 5. The following names (named ranges), formulas and operations are used to create the spreadsheet:

Names:

```
E3          _c
E4          p
E5          s
C08:I08     x
B11:B17     q
C11:I17     payoff
J11:J17     ePayoff
J18         maxEPayoff
```

Formulas:

```
C11:I11     {=IF(q<=x,(p-_c)*q,p*x-_c*q+s*(q-x))}
J11         =SUMPRODUCT(probDist,C11:I11)
```

Operations:

1 To enter the payoff formulas, select range `C11:I17`, type the following formula:
`=IF(Q<=D,(p-_c)*Q,p*D-_c*Q+s*(Q-D))`
and press simultaneously `Shift+Ctrl+Enter`

2 Copy `J10` and paste to `J12:J17`.

Formulas:

```
J18         =MAX(ePayoff)
J19         =INDEX(q,MATCH(maxEPayoff,ePayoff,0))
```

As one can see, lots of names and formula are recycled from the other models.

The most significant and distinct formula is defined in range `J11:J17`. It implements the expected payoff model (13). The optimal solution is exposed in red. It is $Q = 110$ with the expected payoff value, $E[Payoff(110,X)]$ = `$55.74`. It means that setting the demand to 110 will result in an average payoff of `$55.74` over a longer period of time. The student can "play" what-if scenarios with this model by changing the probability distribution of demand *X*. The current distribution is symmetric. It should be interesting to examine asymmetric (skewed) distributions. All it takes is to replace the probabilities in range `C10:I10` (named as `probDist`). Caution: make sure that they add up to 1.

This model is available as an Excel workbook, 4-MaxExpectedPayoff.xlsx, stored in ZIP archive spim2018letkowski.zip (Letkowski, 2018).

CASE 5: A NEWSVENDOR ANALYTICAL MODEL WITH A PROBABILISTIC, CONTINUOUS DEMAND

With a continuous probability distribution, models (2) and (4) can be realized, using Calculus. Here, the students must have understanding of rudimentary Calculus, rules of the Probability Theory and Algebra. The following derivation uses the optimization model (3) (4), the cost based model.

The optimization criterion can be expressed and as a [continuously] probability-weighted average over the domain (0, ∞) of demand *X*:

$$E[C(Q,X)] = \int_0^\infty C(Q,x) f(x) dx \qquad (14)$$

It is an expected value of a function that depends on a random variable, having domain (0, ∞) and continuous density, *f*(*x*).

An expected value of a random variable over domain (0, ∞) is defined as (Spiegel, 1975, p.76):

$$E[X] = \int_0^\infty x f(x) dx \qquad (15)$$

If a function, for example, *g*(*x*), depends on a continuous random variable, having probability density f(x), then its expected value is defined as (Spiegel, 1975, p.77):

$$E[g(x)] = \int_0^\infty g(x) f(x) dx \qquad (16)$$

Formula (14) flows directly from (16).

As shown in (3) the cost function is defined by two functions over exclusive and complementary domains (0, Q] and (Q, +∞), respectively. Thus function (14) has to be expressed in terms of two functions, $c_o(Q-X)$ and $c_u(X-Q)$. Fortunately, as shown in (Khan, 2018) it can be done. In general:

$$E[g(x)] = \int_0^\infty g(x) f(x) dx = \int_0^q g(x) f(x) dx + \int_q^\infty g(x) f(x) dx \qquad (17)$$

Applying this rule to the, the expected value of the cost function can be split at point ***q*** as follows:

$$E[C(q,x)] = \int_0^q c_o(q-x) f(x) dx + \int_q^\infty c_u(x-q) f(x) dx \qquad (18)$$

Before applying the optimization principle to function (18), it needs to be simplified. It takes a few transformations to bring this function to a more convenient differentiable state.

Applying the Distributive Property of multiplication, $a(b+c) = ab+ac$, resulting in $c_o(Q-X) = c_oQ-c_oX$ and $c_u(X-Q) = c_uX-c_uQ$ (Wikipedia-Distribute (2018), function (18) can be expressed as follows:

$$E[C(q,x)] = \int_0^q (c_o q - c_o x) f(x)dx + \int_q^\infty (c_u x - c_u q) f(x)dx \tag{19}$$

Now, using this Distributive Propriety again along with the Summation Rule of Integration (Wikipedia-Sum, 2018), the integration expression in (18) can be broken down as follows:

$$E[C(q,x)] = \int_0^q c_o q f(x)dx - \int_0^q c_o x f(x)dx + \int_q^\infty c_u x f(x)dx - \int_q^\infty c_u q f(x)dx \tag{20}$$

Depending on how the students feel comfortable with the Algebra and Calculus rules, the instructor may wish to use visualization techniques to enhance explanation for the (19) to (20) transformation, for example, as shown in Figure 6.

Having noticed that parameters c_o, c_u, and q are constant (they do not impact the integrated function) and applying constant factor rule in integration (Wikipedia-Constant, 2018) one and further simplified function (19):

$$E[C(q,x)] = c_o q \int_0^q f(x)dx - c_o \int_0^q x f(x)dx + c_u \int_q^\infty x f(x)dx - c_u q \int_q^\infty f(x)dx \tag{21}$$

Figure 7 shows this transformation graphically.

The next transformation is a little more complex. It involves converting the integrals with limits (q, ∞) to expressions, using limits $(0, q]$. The first integral represents a partial expected value of demand X. The second one is a right-tail probability.

Using formula (15) and applying the integral splitting rule (17), the expected value of demand X can be expressed as:

$$E[X] = \int_0^q x f(x)dx + \int_q^\infty x f(x)dx \tag{22}$$

Thus, the partial expected value built in the right-hand side expression (21) can be replaced by the following right-hand side expression:

$$\int_q^\infty x f(x)dx = E[X] - \int_0^q x f(x)dx \tag{23}$$

The right-tail probability is complementary to the left-tail probability (Spiegel, 1975, p.41):

$$\int_q^\infty f(x)dx = 1 - \int_0^q f(x)dx \tag{24}$$

By replacing the (q, ∞) integrals with the right-hand side expressions shown in (23) and (24), the expected cost function (21) gets transformed to the following one:

$$E[C(q,x)] = c_o q \int_0^q f(x)dx - c_o \int_0^q x f(x)dx + c_u \left(E[X] - \int_0^q x f(x)dx \right) - c_u q \left(1 - \int_0^q f(x)dx \right) \tag{25}$$

Now, all the integrals have the same limits.

The next transformation is simple. It uses the Algebra's Distributive Property (Wikipedia-Distribute (2018), function (25) gets expressed as follows:

$$E[C(q,x)] = c_o q\int_0^q f(x)dx - c_o\int_0^q xf(x)dx + c_u E[X] - c_u\int_0^q xf(x)dx - c_u q + c_u q\int_0^q f(x)dx \quad (26)$$

The next transformation utilize the simple Commutative Law, $a + b = b + a$, the above function (26) can be rearranged as follows:

$$E[C(q,x)] = c_o q\int_0^q f(x)dx + c_u q\int_0^q f(x)dx - c_o\int_0^q xf(x)dx - c_u\int_0^q xf(x)dx + c_u E[X] - c_u q \quad (27)$$

The final transformations uses the Reverse Distributive Property of multiplication, $ab + ac = a(b + c)$, resulting in further simplification of the excepted value of the cost function:

$$E[C(q,x)] = q(c_o + c_u)\int_0^q f(x)dx - (c_o + c_u)\int_0^q xf(x)dx + c_u E[X] - c_u q \quad (28)$$

This form of the function is differentiable (Wikipedia- Differentiable, 2018) ready for applying the optimization rule for implementing the optimization criterion (4), minimization of the expected cost. Using the first derivative test (Wikipedia-Extrema, Wikipedia-Calculus, 2018), an equation $E[C(q, x)]' = 0$ can be formulated:

$$\left((c_o + c_u)\left(q\int_0^q f(x)dx - \int_0^q xf(x)dx\right) + c_u E[X] - c_u q\right)' = 0 \quad (29)$$

In order to process this equation, the following differentiation rules are employed (Wikipedia- Differentiation, 2018):

Derivative of a sum: $(f(x) + g(x))' = f'(x) + g'(x)$ (31)

Derivative of a product: $(f(x) \cdot g(x))' = f'(x) \cdot g(x) + f(x) \cdot g'(x)$ (32)

Derivative of an integral: $\left(\int_0^q xf(x)dx\right)' = f(q)$ (33)

Here, the students are encouraged to show all detail, intermediate steps, leading to the following equations:

$$E[C(q,x)]' = (c_o + c_u)\left(\int_0^q f(x)dx + qf(q) - qf(q)\right) - c_u' = 0 \quad (34)$$

$$\int_0^q f(x)dx' = \frac{c_u}{c_o + c_u} \quad (35)$$

Since the left-hand side of the above equation is nothing else as the cumulative probability distribution, $F(x)$, of demand X, the optimum solution, q^*, is given by the following equality:

$$q^* = F^{-1}\left(\frac{c_u}{c_o + c_u}\right) \quad (36)$$

It is amazing, how a complicated expression (18) can be reduced to such a simple equality (36). If demand follows a Normal distribution, $N(\mu,\sigma)$, equation (36) can't be expressed, using a closed-form function (Wikipedia-Form, 2018). On the other hand, an approximate solution can be found, using Normal tables or statistical functions. For example, in Excel, the right-hand side inverse function can be approximated very accurately, using function `NORM.INV()`:

q^* =NORM.INV(c_u/(c_o + c_u),μ,σ) (37)

An interesting exercise for the students would be to calculate the mean (μ) and standard deviation (σ) from the empirical probability distribution of demand X: `{(x,p)} = {(70,0.02),( 80,0.1),(90, 0.22),(100, 0.32),(110,0.22),(120,0.1),(130,0.02)}`, used in the previous section (also see Figure 8). This empirical and discrete distribution is symmetric and approximately bell-shaped. Thus, it is interesting to find out if the optimal solutions for the discrete and continuous distributions are close. The following spreadsheet setup shows how to calculate μ (in cell C13) and σ (in cell C14) from the discrete distribution as well as how to return the optimal solution, q*(in cell C18), for the Normal distribution, $N(\mu,\sigma)$:

Names:

```
E3          _c
E4          p
E5          s
C08:I08     x
C10:I10     probDist
C13         μ
C14         σ
C16         cu
C17         co
```

Formulas:

```
C13         =SUMPRODUCT(x,probDist)
C14         =SQRT(SUMPRODUCT((x-SUMPRODUCT(x,probDist))^2,
            probDist))
C16         =p-_c
C17         =_c-s
C18         =ROUND(NORM.INV(cu/(co+cu),μ,σ),0)
```

Figure 9 shows the optimal solution q^* = **105**. This solution is not far from the one obtained by the previous case model, q^* = **110**, for the discrete distribution. A finer granularity of variable Q may provide a better approximation (the previous model uses 10 as the level of the granularity or detail). Figure 10 shows a version of the discrete model in which the granularity level was set to 5. The revised optimal solution for that model is q^* = **105**, which is identical (after rounding to an integer) to the Calculus based solution.

There is only one more task to complete, namely—compute the expected value of the cost, using function (28) for distribution $N(\mu,\sigma)$ that is for the Normal density function:

$$f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}} \qquad (37)$$

With this density function, the expected cost is expressed as follows:

$$E[C(q,x)]=(c_o+c_u)\left(q\int_0^q\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}dx-\int_0^q x\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}dx\right)+c_uE[X]-c_uq \qquad (38)$$

The first integral is the cumulative probability, $p_q = P(X \le q)$. In Excel, this probability can be returned by function `=Norm.Dist(q, μ, σ, True)`. $E[X]$ is the expected value of demand X. With distribution $N(\mu, \sigma)$, $E[X] = \mu$. The second integral is harder (for the Normal distribution). It represents a partial expected value that is difficult, if not impossible, to express in a closed form. With a substitution of this integral by variable μ_q*, the expected cost for $q = q^*$ can be calculated as follows:

$$E\left[C\left(q^*, x\right)\right] = \left(c_o + c_u\right)\left(q^* \cdot p_q - \mu_{q^*}\right) + c_u \cdot \mu - c_u q^* \tag{39}$$

All the components of this function are known except for:

$$\mu_{q^*} = \int_0^{q^*} x \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx \tag{40}$$

Using Excel, or a programming language (Java, C#, Python, etc.), expression (40) can be calculated numerically with any finite precision. Figure 11 shows an output of an Excel procedure that uses a tabulated Normal distribution for a sequence of Q values, starting at `0`, stopping at `105` (q^*), and incremented by $2\Delta x$ = `2(0.005)`, in order to approximate the right-hand side expression (40). Using this procedure, μ_{q^*} is calculated approximately as:

$$\mu_{q^*} = \sum_{i=0}^{n} x_i \cdot P\left(x_i - \Delta x < X \le x_i + \Delta x\right), \text{ where } x_0 = 0, x_i = x_{i-1} + 2\Delta x, x_n = q^* \tag{41}$$

Points x_i are midpoints, distributed through interval $[0, q^*]$ with the probabilities defined around them half the distance from the neighboring midpoints. The students may experiment with the Δx increment, to find out its impact on the precision with which the integral is approximated.

The following name, formula and operation setup is implemented in Excel:

Names:

```
C18            optQ
F16            cumP_for_optQ
F17            μq
N21            Δx
K23:K10523     xUpTo_optQ
N23:N10523     pUpTo_optQ
```

Formulas:

```
E16            ="P(X ≤ "&optQ&")"
F16            =NORM.DIST(optQ,μ,σ,TRUE)
K23            =L21
K24            =K23+2*dx
L23            =NORMDIST(K23-dx,μ,σ,1)
M23            =NORMDIST(K23+dx,μ,σ,1)
N23            =M23-L23
```

Operations:

1	Copy the formula in `K24` and paste it to range `K25:K10523`
2	Copy the formulas in `L23:N23` and paste them to range `L24:N10523`

Formulas:

```
F17    =SUMPRODUCT(xUpTo_optQ,pUpTo_optQ)
F18    =(co+cu)*(optQ*cumP_for_optQ-μq)+cu*(μ-optQ)
```

The final solution is defined as $q^* = 105$ and $E[C(q^*,x)] = \$4.13$. The students are encouraged to run more What-If experiments in order to determine sensitivity of the optimal solution to changes in the model's parameter values.

This model is available as an Excel workbook, 5-MinExpectedCostViaCalculus.xlsx, stored in ZIP archive spim2018letkowski.zip (Letkowski, 2018).

CASE 6: A NEWSVENDOR SIMULATION MODEL WITH A PROBABILISTIC, CONTINUOUS DEMAND

Simulation is the process of developing a model of a real system and conducting experiments with this model in order to better understand the system's behavior or to evaluate some decisions impacting the system's outcomes (Shannon, 1975). A simulation experiment consists of several simulation runs.

The Newsvendor model, expressed by the profit oriented functions (1) and (2) or by the cost based functions (3) and (4), binds the decision variable, Q, and random variable, X, with the objective functions that serve as optimization criteria. In the context of the Newsvendor problem, one simulation run, R_k, applies the same value of decision variable, $Q = q_k$, while generating randomly a sequence of random demand, X, values ($x_{k(1)}, x_{k(2)}, \ldots, x_{k(n)}$). For each pair of the decision and random variable values, ($q_k, x_{k(j)}$), $k = 1,2, \ldots, m$, $j = 1,2, \ldots, k(n)$, one can compute the payoff value (1) or the cost value (2). The simulation model, shown below, uses the cost function (3) and optimization criterion (4). Thus, the simulation experiment consists of m simulation runs:

$$\mathrm{E} = \{R_1, R_2, \ldots, R_m\} \qquad (41)$$

Each simulation run results is a series of system's outcomes returned by the cost function (3):

$$R_k = \{Cost(q_k, x_{k(1)}), Cost(q_k, x_{k(2)}), \ldots, Cost(q_k, x_{k(n)})\}, k = 1,2, \ldots, m \qquad (42)$$

The goodness (optimality) of the decision variable value, Q, is evaluated by the mean value of the cost function:

$$E[Cost(q_k, X)] = \frac{\sum_{i=1}^{n} Cost(q_k, x_{k(i)})}{n} \qquad (43)$$

Consistent with criterion (4), the optimal solution, among options $q_1, q_2, \ldots, q_m$, is the value that minimizes expected cost (43):

$$q^* : E[Cost(q^*, X)] = \underset{k=1,2,\ldots,m}{Min} \{E[Cost(q_k, X)]\} \qquad (43)$$

Solution (43) represent a value resulting from a brute-force search procedure (Wikipedia-Brute-Force, 2018). One tries each of available options and selects the best one, here –the one that minimizes expected cost function. With more complex models, dealing with continuous decision variables, more refined algorithms can be employed, for example Stochastic Approximation, Gradient Methods, Heuristic Algorithms, etc. (Wikipedia-Optimization, 2018).

In a spreadsheet world, a simulation model could be referred to as a What-If model. With only one decision variable (Q), the simulation experiment can be carried out, using the `Data > What-If Analysis > Data Table` command. Here, the `Data Table` setup (`m`-rows by `n`-columns) takes the instance of the model, $Cost(q_k, x_{k(j)})$, and replicates it in the table by setting row-input to a randomly generated demand values, $X \in \{x_{k(1)}, x_{k(2)}, \ldots, x_{k(n)}\}$, and column-input to the selected decision variable values, $Q \in \{ q_1, q_2, \ldots, q_m\}$. The $Cost(q_k, x_{k(j)})$ model is based on function (3):

$$Cost\left(q_k, x_{k(j)}\right) = \begin{cases} c_o\left(q_k - x_{k(j)}\right) & if \quad x_{k(j)} \leq q_k \quad k = 1,2,\cdots,m \\ c_u\left(x_{k(j)} - q_k\right) & otherwise \quad j = 1,2,\cdots,n \end{cases} \tag{44}$$

Figure 12 shows a spreadsheet model for one-day cost, $Cost(q_k, x_{k(j)})$, where $Q = q_k$. and $X = x_{k(j)}$. This model is developed, using the following Excel settings:

Names:

```
C5     _c
C6     P
C7     s
C8     cu
C9     co
H5     µ
H6     σ
C11    rnd
H11    x
E13    q
E15    dayCost
```

Formulas:

```
C11    =RAND()
H11    =ROUND(NORMINV(rnd,µ,σ),0)
E15    =IF(q<=x,cu*(x-q),co*(q-x))
```

This is a what-if model that shows a single output (cell E15 named as dayCost) for the inventory cost based on the order quantity (cell E13 named as q) and randomly generated demand (cell H11 named as x).

The one-day model, $Cost(q_k, x_{k(j)})$, is replicated to produce experiment E (41), using a 2-way What-If table. The following settings and operations can be used to implement experiment E (Figure 13):

Names:

E17	minAvgCost
E18	optQ
K19:W19	costSample
J21	dayCostRef
K21:W21	qDomain
J22:J70	runNumbers
J21:W70	simExp
K19:W19	costSample

Formulas:

J21	=dayCost (This is a linked copy of the daily cost function defined in cell E15, named as dayCost.)

Operations:

1	In order to fill range runNumbers with sequence 1,2,…,49, type 1 into the first cell of range runNumbers. Next, execute command: Home > Fill > Series > Columns > Step value: 1, Stop value: 49, OK.
2	Fill range qDomain with sequence 70, 75, … , 130: Type 0 into the first cell of range qDomain. Execute: Home > Fill > Series > Rows > Step value: 5, Stop value: 130, OK
3	To generate the simulation experiment (a What-If application) do the following: Select range J21:W70. Since this range is named as simExp, you can get the range by picking name simExp from the Name box. Next, run the DATA > What-If Analysis > Data Table … command. As a Row input cell, select cell q (the name of cell E13). As a Column input cell, select any empty cell (for example J18).
4	Calculate the average cost for each option of the Q value defined in the What-If table (range qDomain).

Formula:

K19	=AVERAGE(K22:K70)

Operation:

5	Copy the formula in K19 and paste to range L19:W19.

Formulas:

```
E17       =MIN(costSample)
E18       =INDEX(qDomain,1,MATCH(minAvgCost,costSample,))
```

The last formula (cell E18 named as *optQ*) finds the optimal solution among *Q* options stored in range K21:W21 (named as *qDomain*). An important aspect of the experiment is not only finding the [approximate] optimal solution but also learning more about the functional relation between the order quantity (decision variable *Q*) and the expected inventory cost, $E[Cost(Q,X)]$. Such a relation exists between ranges K21:W21 (*domainQ*) and K19:W19 (costSample). It can be easily visualized, using an X-Y chart (Figure 14). The following instruction sets up this chart:

Operations:

1	Set up a `Scatter (X, Y)` chart with the `Smooth Lines` pattern.
2	Set `X axis` range to `qDomain (K21:W22)`
3	Set `Y axis` range to `qDomain (K19:W19)`
4	Right-click the chart and pick option `Select Data.`
5	In the left panel, click option `Add.`
6	Set Series `X` values to cell `E18.`
7	Set Series `Y` values to cell `E17.`
8	Select and increase the size of the new data point marker.
9	Add the `Data Label` to this point and format the label to include its `(X,Y)` coordinates.
10	Add titles to the chart and both `X` and `Y` axes.

Notice that the menu options for formatting a chart component can be revealed by pointing to and right-clicking the component.

The chart shows the minimum, average, inventory-cost value of $4.22 for order-quantity 105. Recall that the optimal solution provided by the calculus based procedure is (105, $4.13). Although these solutions look close, the simulation experiment provides a random approximation and should be scrutinized statistically. The average cost is just a point-estimate of the expected cost. Here, the students have an opportunity to enhance the simulation estimation by calculating confidence intervals for the expected inventory cost with respect to order quantities that are equal and close to the current point estimate of the optimal solution. The confidence intervals, as shown in Figure 15, are developed, using the following Excel settings:

Formulas:

```
Z7        =Q19
Z8        =STDEV.S(Q22:Q70)
Z9        =COUNT(Q22:Q70)
Z10       =CONFIDENCE.T($Z$6,Z8,Z9)
Z13       =Z7-Z10
Z14       =Z7+Z10
Z15       =Z14-Z13
```

Operations:

1 Copy the formulas in range `Z7:Z10` and paste them to cells `AA7:AB7`.
2 Copy the formulas in range `Z13:Z15` and paste them to cells `AA13:AB13`.

Since the population standard deviation is unknown, a ***t***-Distribution is applied. Using the significance level, α, standard deviation, s, and sample size, n, the margin of error is calculated as =CONFIDENCE.T(α, s, n). The 95% (1-α) confidence intervals, for Q = 100, 105, 110, are:

$E[Cost(100,X)] \in$ (\$3.41, \$5.79)
$E[Cost(105,X)] \in$ (\$3.55, \$5.64)
$E[Cost(110,X)] \in$ (\$3.56, \$5.16)

It is interesting to note that the calculus based solution (\$4.13) is contained in the three confidence intervals.

As an additional challenge, the students may try to replicate the simulation experiment in order to find out how frequently the near-optimal solutions are generated.

This simulation model is available as an Excel workbook, 6-Simulation(What-If)_MinCost.xlsm, stored in ZIP archive spim2018letkowski.zip (Letkowski, 2018).

CONCLUSIONS

The cases presented in this paper provide a wide range of decision optimization concepts, models, situations and tools. The students are given plenty of opportunities to learn or reinforce their understanding and skills in the areas of Decision Theory, Calculus, Probability, Statistics, Simulation, all implemented, using the Spreadsheet Technology.

The Decision Theory cases help the students better understand consequences of applying different decision strategies. They show how to select the best decision for a given decision strategy for choosing the best inventory order-quantity under uncertain demand.

The Probabilistic case, with discrete distribution of demand, demonstrates the power for the expected-value approach when the domain for the feasible solutions (order quantities) is also discrete.

The Calculus-based case shows an elegant solution assuming that the probability distribution of demand is known. It also addresses limitations of Calculus and suggests how to overcome them, using a numerical-integration technique.

The Simulation case is an alternative to the Calculus case. It is based on a sampling (replication) approach. It shows how to evaluate feasible solutions (order quantities) by replicating model outcomes (inventory cost) for each of the solutions. The best solution is the one that has the lowest average cost. Each of the near-optimal solutions is also described statistically by computing their confidence intervals.

All solution procedures, provided in this paper, utilize spreadsheet formulas, functions and commands. An interesting student project may involve other tools, for example, Python, Java, C#, etc.

It is important to note that there exist other solution models for the Newsvendor Problem. A prominent approach is based on the Incremental Analysis (Anderson, et al. 1995, p. 651) also referred to as Marginal Analysis (Winston, 1994, p. 900). This approach provides the optimal solution that is consistent with the Calculus-based one. The students may wish to study this approach, using numerous Web resources, for example, Chapter 7 (The Newsvendor Problem) provided by (Porteus, 2002).

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APPENDIX

	A	B	C	D	E	F	G	H	I	J
1		The *MaxiMax* Model for the Newsvendor Problem								
2		*Discrete and Uncertain Demand*								
3		*Unit Cost*	*c*	$0.4						
4		*Unit SellingPrice*	*p*	$1.0						
5		*Unit Scrap Value*	*s*	$0.1						
6		*Payoffs for the MaxiMax Strategy*								
7		Order	Demand, *X*							
8		Quantity, *Q*	80	90	100	110	120	130	140	MaxPayoff
9		80	$48	$48	$48	$48	$48	$48	$48	$48.00
10		90	$45	$54	$54	$54	$54	$54	$54	$54.00
11		100	$42	$51	$60	$60	$60	$60	$60	$60.00
12		110	$39	$48	$57	$66	$66	$66	$66	$66.00
13		120	$36	$45	$54	$63	$72	$72	$72	$72.00
14		130	$33	$42	$51	$60	$69	$78	$78	$78.00
15		140	$30	$39	$48	$57	$66	$75	$84	$84.00
16									MaxiMax Payoff	$84.00
17									Optimal Decision, Q	140

Figure 16. Spreadsheet implementation of the *MaxiMax* strategy.

Payoffs							
Order	Demand, *X*						
Quantity, *Q*	80	90	100	110	120	130	140
80	$48	$48	$48	$48	$48	$48	$48
90		$54	$54	$54	$54	$54	$54
100			$60	$60	$60	$60	$60
110				$66	$66	$66	$66
120					$72	$72	$72
130						$78	$78
140							$84

Payoffs							
Order	Demand, *X*						
Quantity, *Q*	80	90	100	110	120	130	140
80							
90	$45						
100	$42	$51					
110	$39	$48	$57				
120	$36	$45	$54	$63			
130	$33	$42	$51	$60	$69		
140	$30	$39	$48	$57	$66	$75	

Figure 17. Doing the payoff function, using two independent formulas.
Left: =(pp-pc)*vQ, Right: =pp*vD-pc*vQ+ps*(vQ-vD).

	A	B	C	D	E	F	G	H	I	J
1		The *MaxiMin* Model for the Newsvendor Problem								
2		*Discrete and Uncertain Demand*								
3		*Unit Cost*	*c*	$0.4						
4		*Unit SellingPrice*	*p*	$1.0						
5		*Unit Scrap Value*	*s*	$0.1						
6		*Payoffs for the MaxiMin Strategy*								
7		Order	Demand, *X*							
8		Quantity, *Q*	80	90	100	110	120	130	140	*MinPayoff*
9		80	$48	$48	$48	$48	$48	$48	$48	$48
10		90	$45	$54	$54	$54	$54	$54	$54	$45
11		100	$42	$51	$60	$60	$60	$60	$60	$42
12		110	$39	$48	$57	$66	$66	$66	$66	$39
13		120	$36	$45	$54	$63	$72	$72	$72	$36
14		130	$33	$42	$51	$60	$69	$78	$78	$33
15		140	$30	$39	$48	$57	$66	$75	$84	$30
16									MaxiMin Payoff	$48
17									Model Decision, q_k	80

Figure 18. Spreadsheet Implementation of the *MaxiMin* strategy.

	A	B	C	D	E	F	G	H	I	J
1		The *MiniMax* Regret Model for the Newsvendor Problem								
2		*Discrete and Uncertain Demand*								
3		*Unit Cost*	*c*	$0.4						
4		*Unit Price*	*p*	$1.0						
5		*Unit Scrap Value*	*s*	$0.1						
6		*Payoffs*								
7		Order	Demand, *X*							
8		Quantity, *Q*	80	90	100	110	120	130	140	
9		80	$48	$48	$48	$48	$48	$48	$48	
10		90	$45	$54	$54	$54	$54	$54	$54	
11		100	$42	$51	$60	$60	$60	$60	$60	
12		110	$39	$48	$57	$66	$66	$66	$66	
13		120	$36	$45	$54	$63	$72	$72	$72	
14		130	$33	$42	$51	$60	$69	$78	$78	
15		140	$30	$39	$48	$57	$66	$75	$84	
16		MaxPayoff	$48	$54	$60	$66	$72	$78	$84	
17										
18		*Regret*								
19		Order	Demand, *X*							
20		Quantity, *Q*	80	90	100	110	120	130	140	Max Regret
21		80	$0	$6	$12	$18	$24	$30	$36	$36
22		90	$3	$0	$6	$12	$18	$24	$30	$30
23		100	$6	$3	$0	$6	$12	$18	$24	$24
24		110	$9	$6	$3	$0	$6	$12	$18	$18
25		120	$12	$9	$6	$3	$0	$6	$12	$12
26		130	$15	$12	$9	$6	$3	$0	$6	$15
27		140	$18	$15	$12	$9	$6	$3	$0	$18
28									*MiniMax Regret*	$12
29									*Decision,* q_k	120

Figure 19. Spreadsheet implementation of the *MiniMax Regret* strategy.

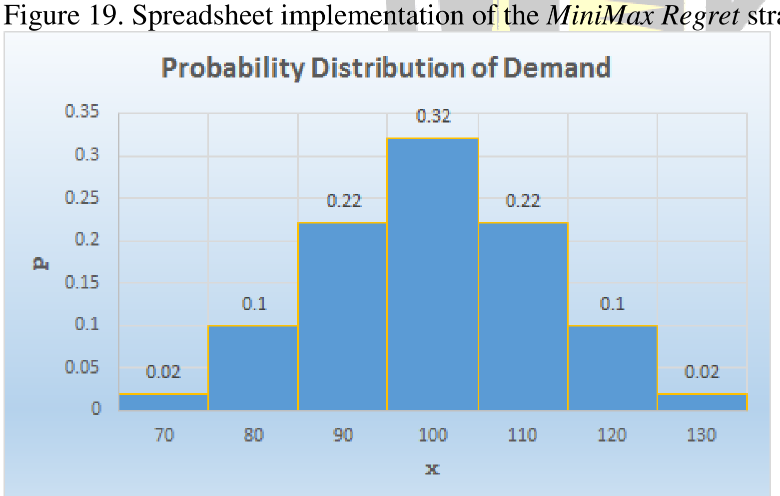

Figure 20. The probability distribution of discrete demand.

	A	B	C	D	E	F	G	H	I	J
1		**The Expected Payoff Model for the Newsvendor Problem**								
2		*A Discrete Case with Known Probability Distribution of Demand*								
3		*Unit Cost*	*c*	$0.40						
4		*Unit Price*	*p*	$1.00						
5		*Unit Scrap Value*	*s*	$0.10						
6		*Payoffs*								
7			Demand, *X*							
8		Order	70	80	90	100	110	120	130	
9		Quantity, *Q*	Probability of Demand, $p(x_i) = P(X=x_i)$							Expected
10			0.02	0.1	0.22	0.32	0.22	0.1	0.02	Payoff
11		70	42	42	42	42	42	42	42	42.00
12		80	39	48	48	48	48	48	48	47.82
13		90	36	45	54	54	54	54	54	52.74
14		100	33	42	51	60	60	60	60	55.68
15		110	30	39	48	57	66	66	66	55.74
16		120	27	36	45	54	63	72	72	53.82
17		130	24	33	42	51	60	69	78	51.00
18									*Max E [Payoff]*	$55.74
19									*Q**	110

Figure 21. Spreadsheet implementation of the Maximum Expected Payoff model.

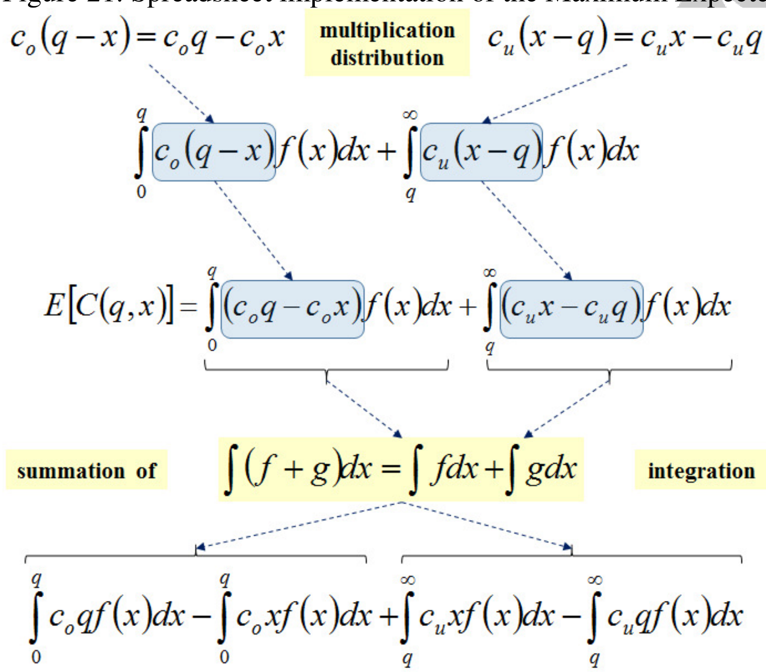

Figure 22.Transforming an integration expression, using distribution of multiplication (Algebra) and summation of integration (Calculus) rules.

$$\int_0^q c_o q f(x)dx - \int_0^q c_o x f(x)dx + \int_q^\infty c_u x f(x)dx - \int_q^\infty c_u q f(x)dx$$

constant factor rule in integration

$$\int cf(x)dx = c\int f(x)dx$$

$$c_o q\int_0^q f(x)dx - c_o\int_0^q x f(x)dx + c_u\int_q^\infty x f(x)dx - c_u q\int_q^\infty f(x)dx$$

Figure 23. Applying the constant factor rule in integration.

F18 =(co+cu)*(optQ*cumP_for_optQ-μq)+cu*(μ-optQ)

	A	B	C	D	E	F	G	H	I
1		The Expected Payoff Model for the Newsvendor Problem							
2		A Continuous Case with Known Probability Distribution of Demand							
3			Unit Cost	c	$0.40				
4			Unit Price	p	$1.00				
5			Unit Scrap Value	s	$0.10				
6									
7			Demand, X						
8			70	80	90	100	110	120	130
9			Probability of Demand, $p(x_i) = P(X=x_i)$						
10			0.02	0.1	0.22	0.32	0.22	0.1	0.02
11									
12		The Expected Demand Values of the Mean and Standard Deviation							
13		μ	100.00						
14		σ	12.649111						
15		The Analytical (Calculus)Solution							
16		c_u	0.6		$P(X \le 105)$	65.37%			
17		c_o	0.3		$\mu_{q^*} \approx$	$60.72			
18		q^*	105	$E[Cost(q^*)]$		$4.13			

Figure 24. The Calculus based solution.

The Expected Payoff Model for the Newsvendor Problem

Discrete and Continuous Cases with Known Probability Distribution of Demand

Unit Cost	*c*	$0.40
Unit Price	*p*	$1.00
Unit Scrap Value	*s*	$0.10

Payoffs

Order Quantity, *Q*	Demand, *X*: 70	75	80	85	90	95	100	105	110	115	120	125	130	Expected Payoff
Probability of Demand, $p(x_i) = P(X=x_i)$	0.013	0.023	0.054	0.082	0.105	0.137	0.172	0.137	0.105	0.082	0.054	0.023	0.013	
70	42	42	42	42	42	42	42	42	42	42	42	42	42	42.00
75	41	45	45	45	45	45	45	45	45	45	45	45	45	44.94
80	39	44	48	48	48	48	48	48	48	48	48	48	48	47.78
85	38	42	47	51	51	51	51	51	51	51	51	51	51	50.37
90	36	41	45	50	54	54	54	54	54	54	54	54	54	52.60
95	35	39	44	48	53	57	57	57	57	57	57	57	57	54.35
100	33	38	42	47	51	56	60	60	60	60	60	60	60	55.49
105	32	36	41	45	50	54	59	63	63	63	63	63	63	55.85
110	30	35	39	44	48	53	57	62	66	66	66	66	66	55.60
115	29	33	38	42	47	51	56	60	65	69	69	69	69	54.87
120	27	32	36	41	45	50	54	59	63	68	72	72	72	53.78
125	26	30	35	39	44	48	53	57	62	66	71	75	75	52.44
130	24	29	33	38	42	47	51	56	60	65	69	74	78	51.00
													Max E [Payoff]	$55.85
													*Q**	105

Figure 25. The Max Payoff Model with discrete distribution and reduced granularity.

μq =SUMPRODUCT(xUpTo_optQ,pUpTo_optQ)

The Analytical (Calculus)Solution

c_u	0.6	$P(X \leq 105)$	65.37%
c_o	0.3	$\mu_{q^*} \approx$	$60.72
q^*	105	$E[Cost(q^*)]$	$4.13

Spreadsheet Integration for μ_{q^*}

x_1	0.00	Δx	0.005
x	$P(X \leq x-\Delta x)$	$P(X \leq x+\Delta x)$	p
0.00	0.000000000000	0.000000000000	0.000000000000
0.01	0.000000000000	0.000000000000	0.000000000000
0.02	0.000000000000	0.000000000000	0.000000000000
104.98	0.652954099740	0.653245971247	0.000291871506
104.99	0.653245971247	0.653537751831	0.000291780584
105.00	0.653537751831	0.653829441340	0.000291689508

Figure 26. Extending the Calculus Model for the continuous distribution with an Excel-based numeric approximation of the partial expected value of demand *X*.

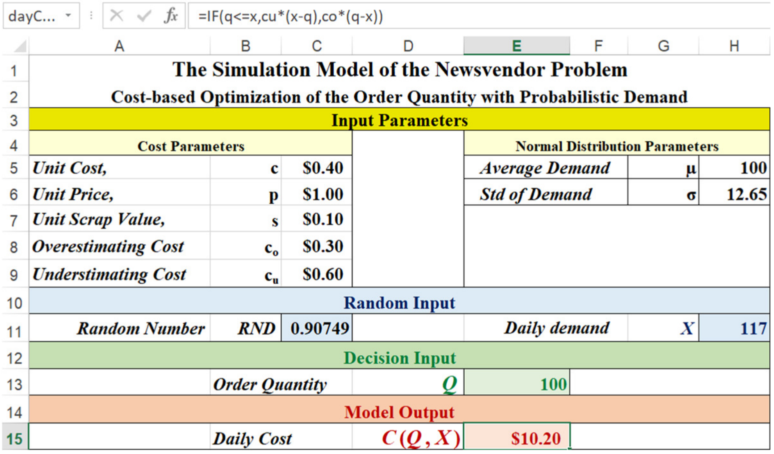

Figure 27. A simulation model for generating a daily cost amount, based on given order quantity *Q* and randomly generated demand *X*.

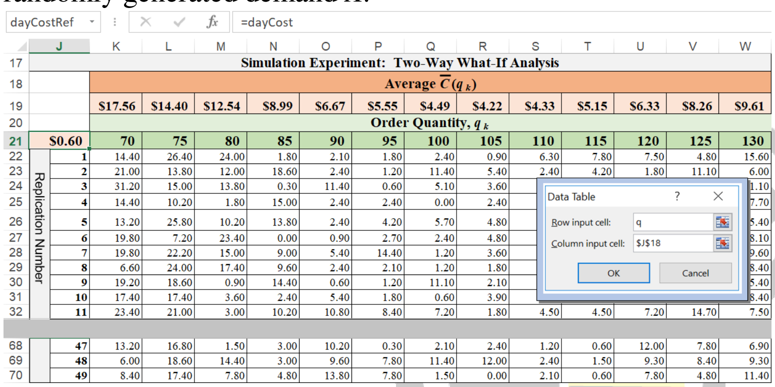

Figure 28. A simulation experiment implemented as an Excel's What-If Table application.

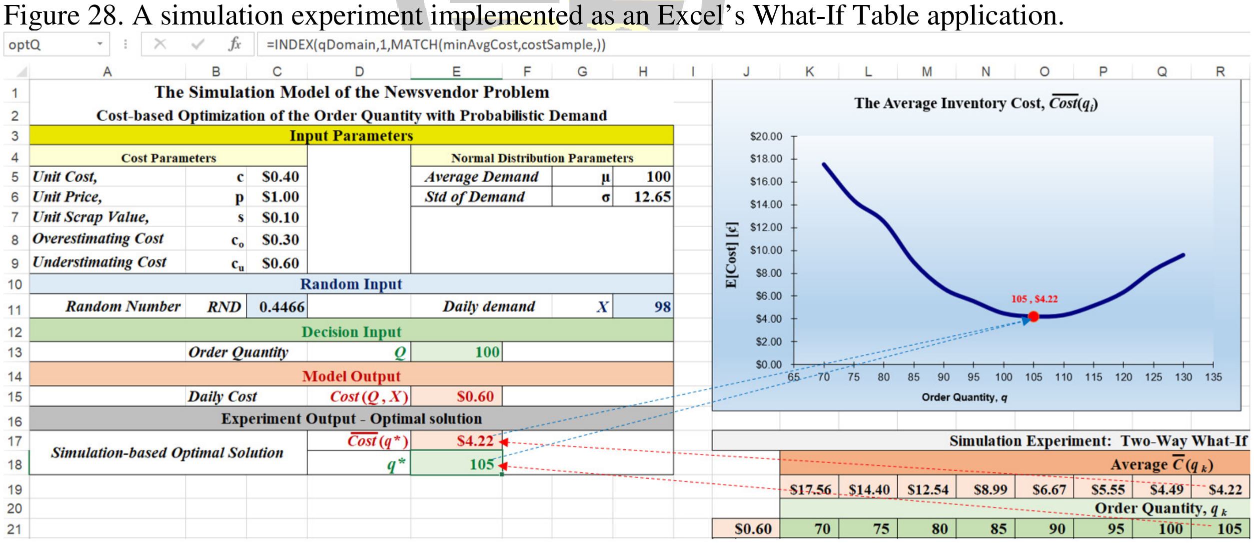

Figure 29. An XY Chart, depicting the simulation experiment outcomes.

AA10 =CONFIDENCE.T(Z6,AA8,AA9)

	X	Y	Z	AA	AB
1					
2			q^*	q^*	q^*
3			100	105	110
4					
5		Expected Cost - Statistical Measures			
6		α	5%		
7		x-bar	$4.49	$4.22	$4.33
8		stDev	$4.12	$3.37	$2.67
9		size	49	49	49
10		error	$1.18	$0.97	$0.77
11					
12		Expected Cost - Confidence Interval			
13		Lower-Limit	$3.30	$3.25	$3.56
14		Upper-Limit	$5.67	$5.19	$5.10
15		Width	$2.37	$1.94	$1.53

Figure 30. Statistical summary, including confidence intervals, for the expected inventory cost, depending on the optimal and near-optimal order quantities.