Number sense of pre-service teachers

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ABSTRACT

The purpose of this study was to a) explore what level of number sense was possessed by pre-service teachers; b) explore what are the effects of a problem-solving-based mathematics course on the number sense of pre-service teachers ; c) compare level of number sense was possessed by pre-service teachers with Tsao's research in 2004. The sample was composed of students in two intact entry-level mathematics sections of a course populated by pre-service teachers. One sixty and eight participants from these two classes completed data collection tasks during the Spring 2011 semester for the study. A problem-solving-based mathematics course was designed to utilize manipulatives, problem solving approaches, and the cooperative learning environment. The focus of the teaching was student thinking and mathematical activity. Results showed that the pretest data of the number domain of Fraction percent of the Number Sense Test was relatively low when compared with Whole Number (42.66%) and Decimal (48.87%). This result was consistent with the finding of earlier studies (Tsao, 2004) that pre-service teachers relied heavily on written algorithms when responding to number sense related questions and that proficiency with exact written computations did not necessarily reflect good number sense. Results also showed statistically significant changes from pretest to posttest were found in the Number Sense Test, and change was significant at the α=0.01 level. Students definitely improved in number sense. It appears that being in a problem-solving-based mathematics course did result in additional necessary practice which helped them to improve their number sense.

Keywords: number sense, problem solving, fraction, pre-service teacher

THE PURPOSE OF THE STUDY

Number sense refers to a person's general understanding of numbers and operations along with the ability and inclination to use this understanding in flexible ways to make mathematical judgements and to develop useful and efficient strategies for managing numerical situations. (Reys & Yang, 1998; McIntosh al., 1999; Tsao, 2004; Yang 2006; Tsao & Rung 2007). It also includes the ability and inclination to use this understanding in flexible ways to make mathematical judgments and develop useful strategies for handling numbers and operations. The National Council of Teachers of Mathematics, in their *Principles and Standards for School Mathematics*, note that number sense is one of the foundational ideas in mathematics in that students (1) Understand number, ways of representing numbers, relationships among numbers, and number system; (2) Understand meanings of operations and how they related to one another; (3) Compute fluently and make reasonable estimates. (NCTM, 2000, p.32).The development of number sense is important in mathematics education. (NCTM, 2000; Yang 2006; Tsao & Lin, 2011) have highlighted that mathematics teachers should have a profound understanding of mathematical concepts and be able to present mathematics as a coherent, integrated and connected endeavor.

However, studies on teachers' number sense (e.g. Ma, 1999; Tsao & Lin 2011) have been limited, and hence this study serves to augment knowledge about teachers' number sense, specifically pre-service teachers' number sense. For instance, Mclnotosh et al. (1992) and Tsao (2004), claimed that high skill in written computation is not necessarily accompanied by number sense. This findings confirm that the content emphasized in mathematics is what is learned and is consistent with the statements (Sowder,1988, p. 227) that "correct answers are not a safe indicator of good thinking" and "teachers must examine more than answers and must demand from students more than answers." some research studies (Tsao & Lin, 2011, Tsao & Rung 2007, Yang, 2002, 2006) have confirmed that teachers play an important role in promoting children's number sense. Tsao (2004) found that pre-service elementary teachers have a gap in their rational number understanding and that they rely on the use of algorithms when approaching non-standard problems. As Ma (1999) stated, "If a teacher's own knowledge of the mathematics taught in elementary schools is limited to procedures, how can we expect his or her classroom to promote the tradition of inquiry mathematics (p. 153)?" It is reasonable to assume that if teachers have no knowledge of number sense, they will have difficulty helping children develop number sense.

A mathematics course designed for pre-service teachers may be the best setting for the study of their understanding of number sense. This study will focus on the pre-service teacher in a problem-solving-based mathematics course that includes the study of number sense. The purpose of this study was to a) explore what level of number sense was possessed by pre-service teachers; b) explore what are the effects of a problem-solving-based mathematics course on the number sense of pre-service teachers ; c) compare level of number sense was possessed by pre-service teachers with Tsao's research in 2004. The institution used in this study has implemented an undergraduate program that requires all prospective teachers to take a three-semester sequence of mathematics courses. These courses are all problem-solving-based. A problem-solving-based mathematics content course was designed to utilize manipulatives, problem solving approaches, and cooperative learning environment. Students actively participate in problem-solving mathematical exploration. The focus of the teaching was student thinking and mathematical activity. The students take part in hands on activities utilizing manipulatives learning mathematical ideas in much the same way elementary school students learn mathematics under a standard-based curriculum.

BACKGROUND OF STUDY

Number Sense

By referring to how number sense was exhibited, Nickerson and Whitacre (2010), number sense can be described as good intuition about numbers and their relationships. Individuals with good number sense tend to exhibit the following characteristics when performing mental computations; sense-making approach, planning and control, flexibility and appropriateness sense of reasonableness. Tsao (2004) found that the use of number sense can assist individuals in their understanding of, and calculation in, mathematics. She reported on these aspects of number sense by studying pre-service teacher education students. The researcher found the pre-service teacher students in this study displayed underdeveloped sense of number, exhibited a preference for using exact written calculations and seldom utilized approaches involving estimation, and desired to follow a set line of reasoning without reviewing the appropriateness of the strategies employed or reasonableness of results obtained. More broadly stated by Sowder (1994), number sense refers to: A well organized conceptual network that enables a person to relate number properties with operation properties. It can be recognized by the ability to compose and decompose numbers and move flexibly among different representations, to compare and order numbers, to use benchmarks to deal with absolute magnitude of numbers, to link numeration operations and relation symbols in a meaningful way, to mentally calculate and estimate using invented strategies, to understand the effects of operations on numbers, and to be disposed to make sense of number (p. 145).

McIntosh et al.(1992) developed a number sense framework based on research and reflection on the literature related to the topic. From the framework, six major components of number sense were identified. See Table 1.

Table 1. Six Components of Number Sense

Number Sense component	Example
Understanding of the meaning and size of number	How does 6/11 compare in size to ½? How do you know?
Understanding and use of equivalent representations of numbers	Show different ways that 3/8 can be represented.
Understanding the meaning and effect of operations	Is 1000÷ 0.98 more or less than 1000? How do you know?
Understanding and use of equivalent expressions	Are 800 ÷0.5 and 800×0.5 equivalent? How do you know?
Flexible computing and counting strategies for mental computation, written computation, and calculators.	Can you multiply 6×99 mentally by using your understanding of numbers and operations?
Measurement benchmarks	Can you estimate the height of a large object? Can you use a benchmark and operation?

Number Sense Related Pre-service Teachers

Research suggests that elementary teachers effect both the achievement and the attitude of students in mathematics. In fact, elementary teachers play an important role in the early mathematical environment for students. Results of Tsao and Lin's (2011) study of expert teachers indicate considerable variability in teachers' knowledge of number sense. Several studies have highlighted that teachers play a key role in helping students develop number sense through well-designed number sense activities and by creating a good learning environment and promoting class discussion (Markovits & Sowder, 1994; Yang & Reys, 2001; Tsao & Rung 2007).

Pre-service teachers' poor performance in fractions can cause serious problems. Pre-service teachers will be teaching mathematics in elementary schools. The misconceptions they exhibit tend to be similar across different representations of rational numbers. The findings of Tsao (2004) and Hungerford (1994) suggest that pre-service teachers exhibit difficulties with rational numbers that may be indicative of a lack of intuitive conceptual understanding of the meaning and properties of the number system. Evidence has shown that pre-service teachers have difficulties with the concept of fractions and the meaning of division of fractions (Ball, 1990 & Tsao, 2004). Tsao (2004) found that pre-service teachers' understanding of the operation of division relied on rules and was unrelated to other mathematical operations. Johnson (1998) conducted a study yielding more evidence that prospective teachers' general number sense and rationale number concept knowledge are inadequately developed. These students, resist looking at mathematics in creative, non-algorithmic ways. Yang (2007) conducted a study yielding more evidence that prospective teachers' general number sense concept knowledge is inadequately developed. Results indicated that about one-third of these pre-service teachers were able to use number sense strategies (such as recognizing the number size, using benchmarks, etc.) and the other two-thirds relied heavily on written algorithms to solve problems. This is consistent with the findings of the earlier studies (Tsao, 2004; 2005), which state that pre-service elementary teachers rely heavily on the written method when responding to number sense related questions. This implies that the performance of pre-service elementary teachers on number sense is low. If we want to improve elementary students' knowledge and use of number sense, then we should try to improve the ability of their future teachers' number sense.

Problem-Solving-Based Classrooms

Acquisition of mathematical knowledge through problem solving and with manipulatives has long been considered to be time-consuming and labor intensive for many classroom teachers who are seemingly overwhelmed with high-stakes testing and unacceptable achievement scores on international measures (National Science Foundation, 2004). This has since encouraged educators to study ways in which teaching and learning occurs in the elementary classroom, particularly when a primary goal is to teach students fluency and flexibility with numbers and strategies for using those numbers (NCTM, 2000). It recommends that pre-service elementary school teachers should be presented with opportunities in their collegiate courses to do mathematics: explore, analyze, construct models, collect and represent arguments, and solve problems. The document further recommends pre-service elementary school teachers discuss concepts, reflect on their thinking, pose questions, answer questions, present logical arguments, and critique the work of other students to help them develop pedagogical content skills necessary for teaching mathematics.

The math course used in this research is taught in a constructivist manner. The focus of the teaching is student thinking and mathematical activity. The mathematics content course is the first course in a three-course sequence. Topics in the curriculum include quantitative reasoning, place value, meanings for operations, children's thinking, standard and alternative algorithms, representations of rational numbers, and operations involving fractions. The purpose of using manipulatives in this math course is the concrete modeling of abstract mathematical ideas. The nature of manipulative use encourages interaction with not only objects but also with people since it usually involves an action on an object. While it is virtually impossible to show a mathematical concept directly by means of a manipulative, it might be possible for students to construct a concept or discover a mathematical relationship through an appropriate use of a manipulative in a meaningful task environment. The students, mostly pre-service elementary school teachers, take part in hands-on activities utilizing manipulatives (McNernery, 1994; Tsao 2004) and learning mathematical ideas in much the same way their future students may learn mathematics. This type of instruction meets many of the goals of the NCTM Professional Standards. As the pre-service teachers experience mathematics with a focus on student thinking and mathematical activity, they are able to construct meaning on their own, leading to a better understanding of mathematics (NCTM,1989, a).

Using mathematical problems has been advocated as a crucial and motivating component of learning and understanding mathematics. Schoenfeld (1992) notes when solving mathematical problems, students develop a deeper understanding of mathematics because it helps them to conceptualize the mathematics being learnt. Heller and Hungate (1985) pointed out that students can be taught to focus specific strategies which related to various problem-solving exercises. They recommend that (a) tacit processes should be made explicit to the students, (b) students should be involved in talking about processes, (c) students should be provided with guided practice, (d) students should learn the components of Polya's strategies, and (e) teachers should emphasize both the qualitative understanding and specific procedures involved in the problem solving process. However, research has not always provided evidence that teaching heuristics has a significant positive impact on the students' problem-solving skills.

Furthermore, Ridlon (2009) found that nine-week periods during two consecutive years, sixth-grade students at the same school were taught identical mathematics content using two different instructional approaches. Year 1 involved low achievers, whereas Year 2 was mixed ability students. The experimental treatment was a problem-centered approach where potentially meaningful tasks were posed to the class and solved in collaborative groups. The groups presented and defended their solution strategies to their peers. A statistically similar control group learned via the traditional explain-practice approach. The teacher demonstrated procedures and students individually practiced what had been illustrated. Regardless of perceived ability level, the problem-centered approach appeared to significantly enhance achievement and improve attitude towards mathematics. Low achievers seemed to gain the most, narrowing the gap between them and their mixed ability peers.

METHODOLOGY

Sample

The population of this study consists of pre-service school teachers at a mid-sized, four-year, state university in a mid-sized town in the Southern Minnesota region. This study involved 68 pre-service teachers from two sections of mathematics content classes/sections the researcher taught during the Spring 2011 semester for the study. Participants were in a teaching credential program, which included 9 credit hours of mathematics education, leading to a K-8 teaching certification. On the first day of mathematics content class, these 68 pre-service teachers were given the Number Sense Test (NST) .Calculator use was not allowed. At the end of the semester (15 week later) the Number Sense Test was re-administered to the three classes by the researcher. Students were given a copy of the NST and instructed not to begin work until told to do so by the researcher. The researcher was provided with general instructions and answer questions from students. Students were asked to obey the rules of this test: timing per item is about 45 seconds and students were told not to spend too much time on any one question. They were given 20 minutes to complete the test.

Instrument

The Number Sense Test (NST) was developed by Tsao (2004) for pre-service elementary teachers. The 25 item NST includes whole number, fraction, and decimal items as well as the four basic operations. According to Tsao (2004), the split-half reliability of the NST is over 0.77 for pre-service elementary school teachers. Figure 4 provides the framework of NST items by number domain and four basic operations. The Cronbach's alpha coefficient reliability for the Number Sense Test (NST) is 081. The Cronbach's alpha coefficient reliability of instrument has demonstrated consistent reliability for measures of internal reliability. Table 2. shows two items.

Table 2. Sample of Number Sense Test Items

1. Without calculating an exact answer, circle the best estimate for: $6\frac{2}{5} \div \frac{15}{16}$	A. More than $6\frac{2}{5}$ B. Less than $6\frac{2}{5}$ C. Equal to $6\frac{2}{5}$ D. Impossible to tell without working it out
2. Which total is more than 1?	**A.** $\frac{5}{11}+\frac{3}{7}$ **B.** $\frac{7}{15}+\frac{5}{12}$ **C.** $\frac{1}{2}+\frac{4}{9}$ **D.** $\frac{5}{9}+\frac{8}{15}$

NST items 15, 19 and 23 were selected from the Number Sense Group test items constructed by Mcintosh, Reys, & Reys (1997). Item 4 was selected from the Second National Assessment of Educational Progress instrument (Carpenter, Corbitt, Kepner, Lindquist, & Reys,1980). The remaining NST items were created by Tsao (2004). Several items are similar to those from the above sources with variation in numbers and operations. Table 3 presents the items contained in the NST.

Table 3. The Framework of Number Domain on the NST

Number Domain	**Number Items**
Whole number	9, 10, 11, 13, 24, 25
Decimals	1, 5, 7, 8, 14, 16, 17, 18
Fractions	2, 3, 4, 6, 12,15, 19, 20, 21, 22, 23

Data Analysis

Each item of the NST is assigned a maximum of 2 points. On items where the subject gave the correct answer, the item will be awarded 2 points. If there is no response or if the response is incorrect, this item will be assigned 0 points. However, items 10, 12, 18, 20 and 24 have a possible point range of 0 to 2 points. For example, items 10 and 12 require the subject to give correct answers and correct explanations. These items are assigned 2 points. If the answer is correct, but the explanation is unclear or if there is no explanation, this item will be assigned 1 point. If both the answer and reasons are incorrect, this item is assigned 0 points. The total possible score of the NST is 50 points. Students' responses were judged to decide whether the characteristics of " number sense " are correctly used by pre-service teachers. One example from the study was coded as follows:

Table 4. One example from the study was coded.

Questions	Responses	Coded Category
How many decimals are there between 2/5 and 3/5?	There are infinite decimals between 2/5 and 3/5. 41/100, 42/100, 43/100, . . . & 44/100 are between 2/5 and 3/5.	This was judged as correct use of the number magnitude

At the end of the semester, data from students who did not provide data for all measures were not used. All four measures were scored and descriptive statistics provided for each student. The paired NST changes scores were calculated and t-tests were performed at the end of the semester to determine if there are any significant changes in the NST scores between any of the observations.

RESULTS

The NST included items representing three number dimensions: Whole Numbers, Decimals, and Fraction numbers. Table 5 displays the percents of correct responses and standard deviations on the NST by number domains for the sixty and eight participants. The pretest data show that the number domain of Fraction 36.90 percent of the Number Sense Test was relatively low when compared with Whole Number (42.66%) and Decimal (47.00%).

Table 5. Mean and Percent of Correct Responses of Pretest on Number Domain Items for NST.

Number Domain	Number of Items	Possible Scores	Mean	Standard Deviations	Percent
NST Whole	6	12	5.12	2.78	42.66
NST Decimal	8	16	7. 52	3.68	47.00
NST Fraction	11	22	8. 12	4.43	36.90

Table 6 displays the mean, percent of correct responses and standard deviation on the NST by number domains for the posttest. The posttest data show the number domain of Fractions as 57.18. The percent of correct responses on the Number Sense Test was relatively low as compared with the Whole Number (61.33) and Decimal (61.19) the percent of correct responses. Table 7 summarizes the t-test results between the mean scores on the NST Number Domain Items in the pretest and posttest. The resulting change in means are displayed in Table 8.

Table 6. Mean and Percent of Correct Responses of Posttest on Number Domain Items for NST

Number Domain	Number of Items	Possible Scores	Mean	Standard Deviations	Percent
NST Whole	6	12	7.36	2.68	61.33
NST Decimal	8	16	9.91	3.51	61.19
NST Fraction	11	22	12.58	4.53	57.18

Table 7.The t-test results on the NST Number Domain Item

	Mean	Std Error	t value
NST	9.101	0.603	15.091*
NST Whole	2.241	0.256	8.753*
NST Decimal	2.392	0.292	8.191*
NST Fraction	4.468	0.342	13.064*

*p<0.01

Table 8. Pre/Post means for the NST Number Domain Item

	Possible scores	Pretest Mean	Posttest Mean
NST	50	20.76	29.85
NST Whole	12	5.12	7.36
NST Decimal	16	7.52	9.91
NST Fraction	22	8.12	12.58

The t-test results indicated that there was a statistically significant difference between the NST mean score of the pretest and posttest (p =0.0001), at the 0.01 significance level. Furthermore, the t-test results indicated that there was a statistically significant difference between the Number Domain Item mean score of the pretest and posttest (p =0.0001), at the 0.01 significance level. Using $\alpha = 0.01$ as the pre-study determined level of testing, students demonstrated significant change in Number Domain Items. As a student's mathematical experiences and knowledge increased, it is reasonable to conclude that the student's number sense capabilities also mature and develop. The increased mathematics knowledge of students led to an increase in number sense performance. This might be due to students learning concepts in a problem-solving-based mathematics course during this semester.

DISCUSSION

Results showed that the pretest data of the number domain of Fraction (36.90) percent of the Number Sense Test was relatively low when compared with Whole Number (42.66%) and Decimal (47.00%). This result was consistent with the finding of earlier studies (Tsao, 2004; 2005) that pre-service teachers relied heavily on written algorithms when responding to number sense related questions and that proficiency with exact written computations did not necessarily reflect good number sense. Results also showed statistically significant changes from pretest to posttest were found in the Number Sense Test, and change was significant at the α=0.01 level. Students definitely improved in number sense. It appears that being in a problem-solving-based mathematics course did result in additional necessary practice which helped them to improve their number sense.

This study supports the findings confirms that the problem-centered instructional approach can encourage the discussion of students and make sense of mathematical learning (Ridlon, 2009; Tsao 2004). This results indicate the focus of teaching is student thinking and mathematical activity, taking part in hands-on activities, utilizing manipulatives, and learning mathematical ideas so that pre-service elementary teachers may learn mathematics. The process of working with manipulatives and hands-on activities help students develop backup strategies that can be used when they become confused with the mechanisms of newly learned strategies or when they want to be certain that computations are indeed correct. As the pre-service elementary teachers experience mathematics with a focus on student thinking and mathematical activities, they were able to construct meaning on their own, leading to a better understanding of mathematics (NCTM, 1989 b).

Raising the awareness of the importance of number sense in the mathematical development of pre-service teachers is essential for mathematics education. Although a major mathematics education reform effort is presently underway, number sense has been incorporated into teacher education programs. If important improvements are to be made in mathematics for pre-service teachers, the development of number sense should become a major focus. This study found that many pre-service elementary teacher subject of research are not ready to be immersed into a curriculum that reflects the vision of less emphasis on paper-and-pencil computation and more emphasis on number sense and mental arithmetic stated in the NCTM Standards. Therefore, specific steps need to be taken in order to assure that future teachers have a proper conceptual understanding of new definitions of computation and number sense as recommended by the NCTM (2000) and have the skills to implement it.

Teachers of the first mathematics methods course of the three course sequence should be aware that their students are sometimes deficient in unexpected areas. The poor performance on the questions involving fractions is an example. One possible reason for poor results in this area is that many pre-service elementary school teachers' knowledge of fractions is rule-based, understanding of fraction content knowledge is very weak. Fractions are a topic that has often caused difficulty for many students. Evidence has shown that pre-service teachers have difficulties with the concept of fractions and have difficulty in explaining fraction so children and why algorithms work, and cannot carry out fractional computation procedures correctly, even when they have correct answers

(Becker & Lin, 2005). More time spent on developing conceptual knowledge of this topic in the required coursework of pre-service elementary teachers should be beneficial to them.

Number sense is a major theme of the NCTM *Principles and Standards for School Mathematics* (2000). The researcher strongly suggests that the teacher education program must include a topic on learning and teaching number sense. In particular, the ability to recognize the relative magnitude of numbers, ability to deal with the absolute magnitude of numbers, ability to link numeration, operation and, relation symbols in meaningful ways, ability to understand the effect of operations, ability to perform mental computation through " invented " strategies that take advantage of numerical and operational properties, ability to use numbers flexibly to estimate numerical answers to computations and to recognize when estimate is appropriate, and a disposition towards making sense of numbers. When pre-service elementary teachers become aware of the importance of using mental arithmetic and number sense, they may then develop the needed strategies necessary to become competent with this idea. This information concerning the background for the changing perspective of computation will also have important consequences for the way pre-service teachers teach and also for those occasions when they have to inform and convince peers, administrators, and parents about the reasons underlying the increasing emphasis on number sense and mental arithmetic. At the same time, mathematics education programs should design more specific number sense activities and integrate them into the pre-service teacher training courses.

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