SAMPLE PERIOD SELECTION WITH VARIABLE ROLLING WINDOWS: SEARCHING FOR BETA STABILITY

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ABSTRACT

One of most relevant problem about time series model forecast use concern estimation errors and connected forecast errors both of them involve a disappointing out of sample forecast performance. Unbiased estimate, is much more crucial in a context in which it is fundamental obtaining performance close to a benchmark measure, as it typically occurs in fund industry managed in indexing mode. From this point of view, it becomes essential finding technicality, in order to perform a more stable beta parameter.

This study is an integration to the paper already showed in the conference Gbfr Costarica 2008, and considers a sample period selection method based on rolling windows with variable length, applicable for a partial solution of estimation error problem for indexing portfolio strategies.

Sample period selection methodology proposed allows to pick, whichever parameter or model can be considered, which sample period you can consider for a parameter estimate, minimizing an error measure (MSE) in a sample period training-set.

The difference with the previous studies resides in the circumstance that the analyses conducted previously did not provide the possibility to vary dynamically the dimension of rolling window (and connected sample period) on the base of some measure of error (es.MSE). Results obtained in confronting an optical of conventional esteem bases on fixed temporal windows with the same esteem base on variable periods of esteem evidence a control of the estimation error and a better stability of a typical indexing measures as Beta.

Keywords: Estimation error reduction, Asset Allocation, Elton Gruber Portfolio Model, Portfolio Choice, Indexing Funds.

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INTRODUCTION

The explosion of the crisis has brought to light a potentially huge problem for all managed savings divisions The financial crisis exploding from the Lehman Brothers touch paper has thrown financial markets into panic and prompted a huge number of savers to exit from their investment positions, to cash in, even if at prices less than those invested at. This has caused a contraction in intermediate volume and a potential element of fear for the continued operation of the asset management industry.

Even in the absence of critical external events, asset managers have shown difficulties because of poor performance of the funds they manage, even when the funds were tied to a stated benchmark.

In these cases has always been difficult to explain to an investor why the financial instrument on which he had placed his savings has not been able to get results even close to the reference index. This lack of performance has led investors to withdraw their savings placed in management and can potentially be a big problem for the continuous operation of many companies that survive on asset management especially in those countries characterised particularly by the presence of smaller companies. It should be noted that many of the companies in the European market, particularly in the Italian, Spanish and French markets are of small size and lack the capacity to survive for long periods of outflow from money invested. Potentially there is a problem for the whole area of asset management, that should the negative trend not invert, the result would be the disappearance of many small companies to the detriment of competition in all the sector.

To avoid a potential default of the whole system, it would be useful for asset managers to invest in the research for asset allocation methods that allow one to not deviate too far from declared benchmark, taken as a reference of the riskiness of an investment as is typically done in the field of financial funds.

The criticalities to which we point have been the subject of numerous studies in particular in the area of American mutual funds (see among others at. Giambona and Golec (2009), Ivković and Weisbenner (2009), Matallin-Saez, (2008), Zingales (2009), and Schachermayer et al. (2009), for a very last review). To try to compensate at least in part for the negative effects of today's financial crisis, we should first try to recover the confidence of savers who entrust savings, often of a lifetime, to management company.

So as a first measure, to exit from this tunnel, we should first try to adopt asset allocation methods that will allow, at least, to not be too distant from a benchmark selected as reference for the risk of the financial investment. Summarize what we are saying has to do with the techniques for estimating parameters necessary to build a portfolio.

One of most relevant problem about time series model forecast use concern estimation errors and connected forecast errors both of them involve a disappointing out of sample forecast performance. The problem of decision making under imperfect knowledge about the relevant parameters has been recognized for a long time. Many of the contributions of economic theory to the study of financial markets involve the use of parameters that are assumed known or estimable by the economic agents. Generally, it is left to practitioners to provide estimates of these parameters which are then directly substituted, usually via a cookbook formula, for the actual parameter values of the theoretical model.

The opinion is widely shared about a direct relationship between care estimate and sample period dimension. Schittenkopf C., Tini P., Dorffner G.(2002), for instance report, for short forecast about financial time series estimate, that estimation error problem could be partially solved by rolling window. This study considers a sample period selection method based on rolling windows with variable length, applicable for a partial solution of estimation error problem.

Instability phenomena aims at it that portfolios estimated efficient ex-ante, would turn out to be sub-optimal ex-post, because of errors made, in estimating input data. That’s why, for a given holding period, risk and return parameters can assume values considerably different, from the estimated ones ex-ante. This instability implies a bias of portfolio weights, where happens because of excessive correspondence to variations in starting parameters.

There is a plentiful literature, that was dealt with the topic of error estimation, in the space of 50 years and has often stressed how optimization proceedings at the bottom of the various model turn out to be particularly often to the risk to make errors in input parameter estimation.

It is clear that the asset manager, with the object of development of the asset allocation phases, cannot ignore the problem of the consequences due to error estimation, therefore it is necessary to fix a procedure which is able to reduce the instability problem. In order to pursue the aim of reducing the instability problem, this study suggest a comparison between a naïve sampling methods of estimation period, carried out through variable rolling windows, and a common procedure which lies in considering a fixed period, usually 52 weeks.

The plan of the article is as follows. In Section I, we discuss the Variable Rolling Windows estimation methodology that arise when the relevant parameters. We then apply this approach in Elton Gruber portfolio model context to a set of financial data. In Section III, we summarize the main conclusions of the study.

1. VARIABLE ROLLING WINDOWS ESTIMATE

At the end of 90's Richard Michaud and Robert Michaud invented and patented Resampling introducing the new concept of Resampled Efficiency described in Michaud. (1998) that has received a certain regard by literature. Resampling is a method used in portfolio modelling to try to obtain better out of sample performance for given input model parameters.

In the real world, where the possibility of estimating errors for future model forecasts certainly exist, it is necessary to consider the error component in building portfolios. Resampling does this by recombining the input parameters required for a portfolio model. Unbiased estimate, is even much more crucial in a context in which it is fundamental obtaining performance close to a benchmark measure, as it typically occurs in fund industry managed in indexing mode. In this work we try to present a new heuristic approach named variable geometry estimator defined from the concept of variable rolling windows.

In rolling windows context, full sample period is taken to pieces in a set of fixed dimension periods. Variable rolling windows involve in considering a variable time domain within p periods (p =1,…n), number of periods of a generic rolling window, such as to return changeable rolling window time dimension.

Data and sample statistics are valued for every sub-sample periods part of a generic rolling window.

For every set of estimated parameters forecast MSE is computed for the first useful period (we treat with uniperiodal validation set), let be $(T_B+k)–p+1$, (T_B date of beginning of computational procedure and $k=1,...(T- T_B)$), in this way we obtain a number (p) of realized MSE, equal to sub-sample periods in a variable rolling window $(p =1,…n)$ for every parameter taken into account.

Sample period selection methodology proposed (Fig.1) allows to pick, whichever parameter or model can be considered, which sample period you can consider for a parameter estimate, minimizing an error measure (for example MSE) in a sample period training-set. The sample period, among the p of the rolling windows, to which minimum is associated MSE for $(T_B+k)–p+1$ is selected as sample interval on which an estimate for $(T_B+k)–p+2$ is based (test-set).

As first step procedural it is necessary therefore to establish the dimension of the rolling window in the within the entire one sample period. Once established its dimension you proceed with the following iterative procedure: the MSE values are estimated, in a variable sample period for $(T_B+k)–p+1$, and it is determined the sample period (among the p periods) for which minimum MSE has been recorded for a forecast one step ahead (validation set).

The temporal amplitude of the optimal window of esteem is used in order to operate one forecast to $(T_B+k)–p+2$, the forecast data obtained is recorded (test set). Then the sliding window is shifted one step ahead. The models are estimated and evaluated on the second time window and so on for all the sub-sample periods (training-set equal to $T-T_B+max(p)$ and validation-set equal to $T-T_B$ until the completion of the total sample period.

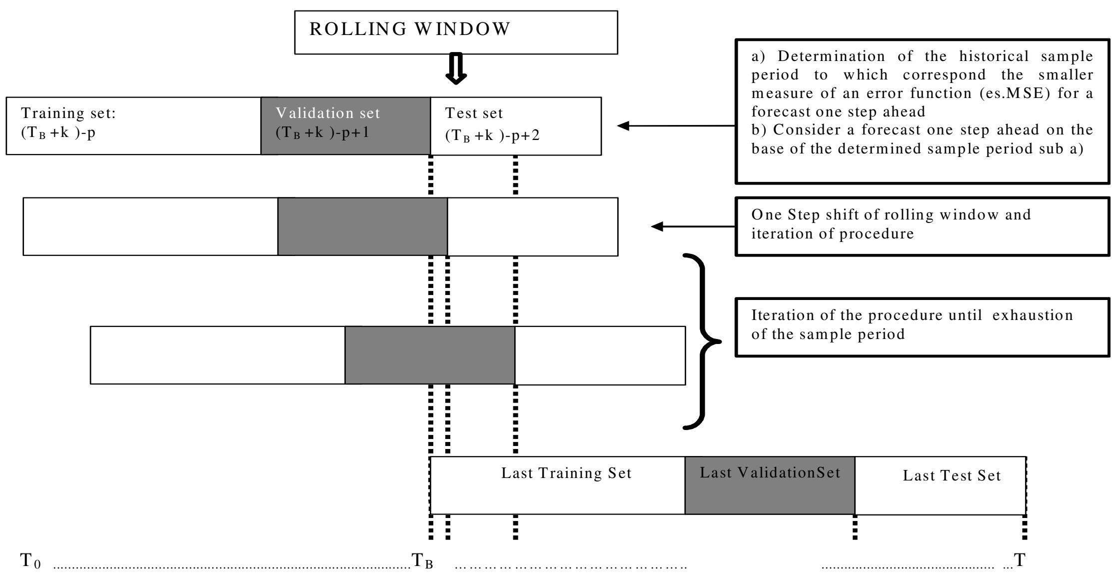
In particular, the test sets are not overlapped and the forecasts can be joined in order to form a large series of out-of-sample profits. In such way it is attempted to find, whichever parameter or model can be considered, a better temporal stability of the esteem to employ in the prevision processes in time series context.

The crucial point consists in verifying if an approach that considers a fixed period of esteem, wide spread in time series modelling, is subject to the production of errors that are characterized by a higher MSE, and for some single considerably greater errors as order of magnitude regarding how much a method of sampling based on rolling windows with variable length can produce.

The difference with the previous studies resides in the circumstance that the analyses conducted previously did not provide the possibility to vary dynamically the dimension of rolling window (and connected sample period) on the base of some measure of error (es.MSE).

The operational rule for generating parameter estimates is not more complicated than a standard resampling approach, and as such it can be easily adopted by practitioners and researchers.

Figure 1: Variable Length Rolling Window



This figure shows the Operational Logic of the Variable Length Rolling Window of Esteem

2. DATA AND RESULT

In order to apply the method we have the following procedure:

1. on the account of the input parameters related to each financial asset, we have to calculate the portfolio return and beta for each test period considered in the range between 15 and 45 weeks.
2. We have to calculate the combinations, of risk and return, of each portfolio would have performed, if the asset class course had been equal to that forecast from the simulation based on the sample period, corresponding to the test period, described in point 1.
3. The optimal test period is the one which corresponds the portfolio obtaining with beta more proximum to 1, for the second test period.

Taking practical cue from the estimate connected to the Asset Allocation operative context, the method of sampling based on variable Rolling windows has been applied and verified (size of the training set used to estimate the model parameters by minimization of an error function (MSE) is one week., size of the validation set is one week too) in confronting the former rendering ex-post return obtained with those ex-ante promises, for blue chips equity of the financial stock indexes Ex50, SpMib40 and Sp100 (period from 11-27-98 to 12-09-06, weekly observation considered) for Elton Gruber portfolio model.

This model has been developed under the assumption of perfect knowledge about the relevant parameters, but in empirical applications the standard procedure is to substitute the sample estimates for the true but unknown parameters. This article advances a practical procedure of parameter estimate for use in the presence of estimation risk. With developments in computer technology it will be even more easy applying this estimation techniques. Table 1 reports summary statistics for Stock Index Market returns over all the sample period taken into account.

It is generally accepted that returns on tradable assets are predictable, and that a significant source of this predictability is the time-varying compensation that investors require for accepting a risky payoff.

Results obtained in confronting an optical of conventional esteem bases on fixed temporal windows (52 weeks) with the same esteem base on variable periods of esteem (variable windows in a range between 1 and 52 weeks) a control of the estimation error and better results for Rap measures out-of-sample. All applications are carried without considering transaction costs connected to periodic portfolio settlement.

The main evidences about dynamically Elton Gruber portfolios of the carried out application (period by period settlement equal as previous application to a one week) starting from an indexing context, that is properly of those asset management schemes seeking for catch a declared benchmark, they can be synthesized in 5 points (see Tables from 2 to 4).

1. Considering that our main purpose is not to show the largest redditivity of variable geometry portfolios but to verify that, with input parameter estimation errors, the naive sampling procedure exhibit risk-return combinations with better stability. Than a better risk adjusted performance can support the aforesaid stability. We can refer to the MSE related to the full sample period as the first synthetic standard measure of risk-return combinations with better stability. The average of the amount of square difference between the forecast beta and performed beta is 43% lower for Ex50 index in the case of combination valued through the naïve sampling method. The foresaid average is 63% lower for the SpMib40 index and quite 50% for Sp100 Index..

2. The matured income of dynamic portfolio, based on he naïve sampling method of variable windows linked to beta equal to 1, is 16% higher for Ex50 index instead it is 5% lower for the SpMib40 index. Relating to the Ex50 index combination, the gap, between the average return obtained with two methods, is about 80%, instead the same measure is about 16% and 1% lower for the SpMib40 index and Sp100 Index respectively.
3. We have to stress the importance of verifying the stability of the gained portfolio combinations relating to the application with target function restrained to beta equal to 1. Therefore it is crucial to verify the variability indexes in the long runt too. So like as already done in the previous application, we have considered a risk measure, in this case, the average standard deviation as measure of the result variability, calculated on three different periods, at 1 year, 3 and 5 years, also in order to stress the stability of the result. The variable geometry estimators imply, for two considered indexes, SpMib40 and Ex50, and for all the considered periods, volatility limitation control in term of standard deviation. In particular the reduction is estimated, according to the considered period, about between 1 and 3% for the SpMib40 index, and about between 3 and 4% for the Ex50 index. Standard deviation containment is bounded between 3 and 6% for Sp100 Index.
4. In relation to the rap measures, for the applications with consider constraints on the portfolio, so that to build asset schemes with try to be closer to a reference index, the best measures is the Information Ratio (IR), which consider the average of track error, that is the differential on average between built portfolio and reference index, and the Tev which measures Tem variability. The portfolio schemes with beta constrained equal to 1, obtained with variable rolling windows, compare the ones produced with fixed windows, imply an IR about once and a half higher for Ex50, and about 15% greater for SpMib40 portfolio combinations. But the kind of this extra-performance is different for portfolios produced by the two indexes: in fact with regard to Ex50, you can record a large increase of Tem, against a little reduced Tev, while in relation to the SpMib40 the larger value of the IR implies a Tem increase (about 2%) and a Tev decrease (about 11%). Tev measures at one and three years describe in a better way the greater stability of the portfolio schemes produces thanks to the naïve sampling methods, in fact for the Ex50 index the Tev decrease, compare to fix window combinations, is contained between 1 and 2%, for SpMib40 the containment is between 1 and 4%. For the same application carried out from Sp100 Index sample, from results showed in table 3 it can be noted a reduction for tracking error volatility for all time span taken into account, an increasing in average term of Track error coupled with a lower volatility of the same variable respect to fixed time window estimate period (52 weeks). This last two evidences lead to a quite dramatic increase in term of Information Ratio.
5. A look to the Downside Risk (DSR), enables to give more qualifications and partially confirms best performances due to estimations through naïve sampling methods. The Dsr estimated at 1 and 3 years for both considered indexes produces less variability in the left part of the distribution of the realized returns, especially for the combinations based on the Ex50 index, the reduction is between 1 and 4%, while for the combinations based on the SpMib40 index the reduction is between 1 and 6%, confirming that the method is effective on the average also related to very negative deviation from the average of the return. Also Sp100 Index result confirm this good capabilities of this heuristic approach to allowance a certain reduction in making large error in parameter estimate.

Table 1 Index Returns – Descriptive statistics

	Sp100	Ex50	Spmib40
Average	0.00093	0.00143	0.00147
Standard Error	0.00117	0.00140	0.00141
Median	0.00057	0.00275	0.00369
Standard Deviation	0.02513	0.02999	0.03021
Sample Variance	0.00063	0.00090	0.00091
Kurtosis	2.10645	2.58117	7.87043
Skewness	-0.19825	-0.03969	0.48027
Sample Return Interval	0.19648	0.25702	0.35037
Min	-0.11228	-0.11594	-0.13674
max	0.08420	0.14108	0.21363

In this table we report descriptive statistics, for all index returns based on whole sample period 11-27-98 to 12-09-06.

Table 2 Ex 50 Result– Indexing Mode (Beta equal to 1)

	52 weeks	**Variable Rolling Windows**	**Extra-performance**		**Benchmark**
	(a)	**(b)**	***(b) - (a)** ****(b)/(a)-1**		
Total Return	4%	20%	16%	*	-7%
Average Return	0,053%	0,095%	80,379%	**	0,020%
Dev..St. (monthly-1 year)	0,0217	0,0207	-0,0442	**	0,020
Dev..St. (monthly-3 year)	0,0187	0,0180	-0,0385	**	0,017
Dev..St. (monthly-5 year)	0,0272	0,0262	-0,0287	**	0,0276
Tem	0,0003	0,0007	1,5877	**	
Tev	0,0421	0,0420	-0,0028	**	
Information Ratio	0,0063	0,0161	1,5499	**	
Tracking Error Volatility (1 year)	0,0296	0,0289	-0,0219	**	
Tracking Error Volatility (3 year)	0,0253	0,0251	-0,0055	**	
Downside Risk (1 year)	0,0398	0,0383	-0,0369	**	
Downside Risk (3 year)	0,0634	0,0630	-0,0063	**	
Mse	0,0068	0,0039	-0,4294	**	

In this table we report results, for the comparison between variable rolling windows estimator and estimate method based on fixed sample period (52 weeks) for Ex50 Index in a sample period 11-27-98 to 12-09-06.

Table 3 SpMib40 Result– Indexing Mode (Beta equal to 1)

	52 weeks	Variable Rolling Windows	Extra-performance		Benchmark
	(a)	(b)	*(b) - (a) **(b)/(a)-1		
Total Return	30%	26%	-5%	*	11%
Average Return	0,113%	0,0952%	-16%	**	0,065%
Dev..St. (monthly-1 year)	0,0192	0,0187	-0,0270	**	0,015
Dev..St. (monthly-3 year)	0,0173	0,0168	-0,0279	**	0,014
Dev..St. (monthly-5 year)	0,0243	0,0239	-0,0135	**	0,022
Tem	0,0005	0,0005	0,0240	**	
Tev	0,0132	0,0117	-0,1133	**	
Information Ratio	0,0342	0,0395	0,1548	**	
Tracking Error Volatility (1 year)	0,0094	0,0091	-0,0376	**	
Tracking Error Volatility (3 year)	0,0084	0,0083	-0,0057	**	
Downside Risk (1 year)	0,0269	0,0252	-0,0627	**	
Downside Risk (3 year)	0,0486	0,0482	-0,0084	**	
Mse	0,0044	0,0016	-0,6278	**	

In this table we report results, for the comparison between variable rolling windows estimator and estimate method based on fixed sample period (52 weeks) for SpMib40 Index in a sample period 11-27-98 to 12-09-06.

Table 4 Sp100 Results– Indexing Mode (Beta equal to 1)

	52 weeks	Variable Rolling Windows	Extra-performance		Benchmark
	(a)	(b)	*(b) - (a) **(b)/(a)-1		
Total Return	46%	45%	-1%	*	-18%
Average Return	0.146%	0.01%	-0.9005	**	-0.042%
Dev..St. (monthly-1 year)	0.018	0.0169	-0.0612	**	0.014
Dev..St. (monthly-3 year)	0.018	0.0174	-0.0333	**	0.015
Dev..St. (monthly-5 year)	0.0213	0.0202	-0.0540	**	0.0207
Tem	0.0005	0.0006	0.2034	**	
Tev	0.0277	0.0269	-0.0289	**	
Information Ratio	0.0181	0.0223	0.2358	**	
Tracking Error Volatility (1 year)	0.0195	0.0190	-0.0256	**	
Tracking Error Volatility (3 year)	0.0169	0.0167	-0.0089	**	
Downside Risk (1 year)	0.0334	0.0318	-0.0480	**	
Downside Risk (3 year)	0.0560	0.0556	-0.0071	**	
Mse	0.0056	0.0028	-0.5089	**	

In this table we report results, for the comparison between variable rolling windows estimator in a indexing fund management context and estimate method based on fixed sample period (52 weeks) for Sp100 Index in a sample period 11-27-98 to 12-09-06.

3.CONCLUSION

This study represent an integration to the paper already showed in the conference Gbfr Costarica 2008, and considers a sample period selection method based on rolling windows with variable length, applicable for a partial solution of estimation error problem for indexing portfolio strategies. Where the Heuristic methods under test are able to obtain the containment of estimation error, they will allow better performance for asset management with benchmark stated, as are the majority of the funds traded in European Financial Markets, allowing for greater retention of savers and to prevent the outflow of monetary resources by investment funds for the benefit of the asset management industry.

First of all results obtained in confronting an optical of conventional esteem bases on fixed temporal windows with the same esteem base on variable periods of esteem evidence a control of the estimation error and a better stability of a typical indexing measures as Beta. In particular, on the sample examined, the average of the amount of square difference between the forecast beta and performed beta is 43% lower for Ex50 index in the case of combination valued through the naïve sampling method, and quite 50 and 63% lower for Sp100 and SpMib40 index respectively.

Moreover the variable geometry estimators imply, for all considered indexes, and for three sub-sample periods, 1 year, 3 years and 5 years, evidence of volatility limitation control in term of standard deviation. In particular the reduction is estimated, according to the considered period, about between 1 and 3% for the SpMib40 index, between quite 3 and 4% for the Ex50 index and bounded in 3 and 5% for Sp100 index.

To summarise the evidence obtained, it is clear that the approach of Variable Geometry estimators is fertile ground for developing new solutions to reduce the error estimation problem and for the production of better performance – for the benefit of savings customers, and in a broader view of the entire financial system – especially in those countries characterized by the presence of numerous small companies. In this article we have only scratched the surface of the question of decision making in the presence of estimation risk via heuristic approach. Considerably more examination of this issue is warranted, both from the perspective of theoretical portfolio models, and at the more fundamental level of practical portfolio management.

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